Absolute Magnitudes Derived Using the Statistical Parallax Method

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ABSTRACT

I review statistical parallax absolute magnitude determinations which employ data from the *Hipparcos* satellite for RR Lyrae and Cepheid variables, and for several other stellar classes. Five groups have studied the RR Lyrae stars, and the results are reassuringly consistent: $M_V(RR) = 0.77 \pm 0.13$ mag at [Fe/H] = –1.6 dex. Extensive Monte Carlo simulations showed that systematic errors are small ($\sim$0.03 mag or less), and corrections for them were applied in the above result. The RR Lyrae result is thus very secure. A statistical parallax study of Cepheids found the Period–Luminosity zero-point to be considerably fainter than studies based on *Hipparcos* trigonometric parallaxes. The distance modulus of the Large Magellanic Cloud derived from this zero-point is in excellent agreement with that derived using the RR Lyrae result. I discuss why the statistical parallax absolute magnitude calibrations differ with some other RR Lyrae and Cepheid calibrations.

Subject headings: statistical parallax; variable stars; RR Lyrae stars; Cepheids

1. Introduction

Statistical parallax is a primary method for determining the mean absolute magnitude, M_V, of a set of stars. That is, the absolute magnitude is determined directly from observables like proper motions and radial velocities, and does not depend on absolute magnitude scales derived for other types of stars for its calibration. In this sense, it is akin to trigonometric parallax in its fundamental contribution to our understanding of the cosmic distance scale.

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Simply stated, statistical parallax works by balancing two measurements of the velocity ellipsoid of the stellar sample.[2] The first measurement is obtained from the stellar radial velocities alone, and is independent of the stars' distances. The second measurement is obtained from the stars' proper motions, and thus is distance-dependent. The velocity ellipsoids are balanced through a simultaneous solution for a distance scale parameter. While this may seem to be a complicated procedure, it employs a model of stellar motions in the Galaxy which has been extremely well tested by countless observational studies of stellar kinematics over the last half century.

The statistical parallax method possesses several other strengths which make it an integral part of the cosmic distance ladder. First, the astrometry required for statistical parallaxes, proper motions, can be determined with smaller relative errors than those of trigonometric parallaxes for stars at a given distance (e.g., a sample of stars from the *Hipparcos* Catalogue (ESA 1997) at $d \approx 500$ pc have $\sigma_\mu/\mu \approx 0.13$ while $\sigma_\pi/\pi \approx 0.62$). Thus, for stars like RR Lyrae and Cepheid variables which are poorly represented near the Sun, statistical parallax becomes more attractive than trigonometric parallax for determining mean absolute magnitudes. Consider the RR Lyrae stars in the *Hipparcos* Catalogue. With one exception, all have $\sigma_\pi/\pi \geq 0.3$ (Fernley et al. 1998). While a representative absolute magnitude may be recovered using careful statistical treatments, the resulting errors in M_V are large, $\sim$0.3 mag (Tsujimoto et al. 1998, Luri et al. 1998). Another attribute of the statistical parallax method is that it does not rely upon model atmospheres, color-temperature calibrations, mass-metallicity relations, stellar evolution models, or their associated simplifications (e.g., convection physics) and assumptions (e.g., helium and light-metal abundances).

In this chapter, I review the recent results involving statistical parallax solutions which use *Hipparcos* data. In §2 I review briefly the development of the modern statistical parallax method, and discuss the various algorithms currently in use. Most of the statistical parallax studies which employ *Hipparcos* data are for RR Lyrae stars. Since these stars are critical to establishing the distance to the Large Magellanic Cloud, and hence the zero-point of the extragalactic distance scale, I focus attention on them in §3. In §4 I discuss one statistical parallax study of Cepheid variables, and in §5 I highlight some results for other types of stars. I summarize the current status of results from the statistical parallax method and provide some thoughts on its future application in §6.

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[2]A velocity ellipsoid consists of three components of bulk motion (U, V, W), their dispersions (σ_U, σ_V, σ_W), and the covariances (C_{UV}, C_{UW}, C_{VW}).

2. Statistical Parallax Generalities

Popowski & Gould (1998a) summarize the distinction between the classical methods of secular and statistical parallax,[3] and how Murray (1983) and followers integrated them into a generalized method which I shall hereafter refer to simply as "statistical parallax". This modern method involves a simultaneous solution for the nine parameters of the velocity ellipsoid plus a distance scaling parameter which relates the observed proper motions to their tangential velocities. A maximum likelihood method is used in the solution to avoid the simplifications adopted by early studies which employed linear least-squares techniques. In some algorithms, additional parameters such as the intrinsic dispersion in the distance parameter (and thus M_V) are included in the solution.

There appear to be about five different statistical parallax algorithms currently in use (see §3). It is difficult to determine exactly how independent the different algorithms are, since they share a common developmental history and employ similar kinematic models and maximum likelihood formulations. However, the methods do show clear differences in such details as the numerical techniques used to maximize the likelihood function and how the uncertainties in the derived parameters are estimated. Also, some algorithms include additional features, such as automatic rejection of outliers (Heck 1975). The algorithm described by Luri et al. (1996) extends this approach by producing separate solutions for distinct groupings it identifies in the parameter space of $(M_V, U, V, W, \sigma_U, \sigma_V, \sigma_W, Z_0)$. This algorithm also models the spatial distribution of each grouping with an exponential disk, and solves for the scale height, Z_0. It also offers an option for modeling the observational selection effects inherent in the stellar sample with additional free parameters such as an apparent magnitude limit. Both the Luri et al. (1996) and Popowski & Gould (1998a) algorithms include a coordinate rotation matrix enabling the bulk velocities and dispersions to be computed in the local frame of reference of each star, (π, θ, z), rather than the Sun-oriented (X, Y, Z) frame.[4] Though the effect of neglecting the rotation is generally small (e.g., §4.3 of Layden et al. 1996) it is worth performing. The Popowski & Gould (1998a) algorithm also includes a treatment of Malmquist bias, and analytic expressions for the uncertainties in each derived parameter. Thus, the algorithms of Luri et al. (1996) and Popowski & Gould (1998a) include some potentially important improvements on previous

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[3]Classical secular parallax balances the three bulk motions (U, V, W) as determined by radial velocities and proper motions, while classical statistical parallax balances the three velocity dispersions $(\sigma_U, \sigma_V, \sigma_W)$ and the three covariances (C_{UV}, C_{UW}, C_{VW}).

[4]That is, the velocities are reported in cylindrical coordinates, (V_π, V_θ, V_z), rather than rectilinear coordinates (U, V, W).

statistical parallax algorithms.

The comprehensive, three paper series by Popowski & Gould is notable for several reasons. First, they present several very instructive discussions which show how the statistical parallax method transforms the input data into the output parameters, and how observational errors, their mis-estimation, and other potential biases affect the solution (Popowski & Gould 1998a, hereafter PG98a). Second is their analytic expression for the relative error in the distance scaling parameter; of particular interest is its dependence on sample size and quality of proper motions (PG98a; discussed here in §3). Third is their development of a hybrid statistical parallax method whereby large samples of stars which do not have proper motions can contribute to the determination of M_V provided they are from the same kinematic population as the set of stars in question (Popowski & Gould 1998b, hereafter PG98b; also see §3). Finally, they showed that attempting to constrain meaningfully the intrinsic scatter in M_V of the RR Lyrae stars or the slope of the RR Lyrae metallicity-luminosity relation using statistical parallax is futile without vastly larger samples of stars.

3. Applications to RR Lyrae Variables

3.1. Pre-*Hipparcos* Work on RR Lyrae

Layden et al. (1996) presented an extensive pre-*Hipparcos* statistical parallax solution for RR Lyrae stars which provides a useful point of comparison for later work. They began their analysis by using their newly compiled observational data and an assumed RR Lyrae absolute magnitude, $M_V(RR)$, to separate the kinematically distinct halo and thick disk components of the sample. The motivation was that mixing these populations could bias the simultaneous solution of kinematics and luminosity. They used ground-based proper motions together with the statistical parallax algorithm of Hawley et al. (1986; based on Murray's (1983) formulation) to obtain $M_V(RR) = 0.71 \pm 0.12$ mag for 162 halo stars with a mean [Fe/H] of –1.61 dex, and $M_V(RR) = 0.79 \pm 0.30$ mag for 51 thick disk stars with $\langle[\mathrm{Fe/H}]\rangle = -0.76$ dex. They performed extensive Monte Carlo simulations to ensure that the statistical parallax solutions and their estimated errors were in good agreement with the known input samples. They also noted some biases which potentially affected their results at the 0.01–0.04 mag level (e.g., the adopted value of the dispersion in the distance scaling parameter, Galactic coordinate rotations, etc).

3.2. *Hipparcos* Work on RR Lyrae

Clearly, the biggest recent change in RR Lyrae statistical parallax analyses has been the advent of high precision *Hipparcos* proper motions (2–3 mas yr^{-1} random errors, 0.25 mas yr^{-1} systematic; Tsujimoto et al. 1998). Figure 1 shows the differences between the *Hipparcos* proper motions (μ^H) and those used by Layden et al. (1996) (μ^{L96}) as a function of μ^H for both Right Ascension and Declination directions. No simple systematic differences are evident,

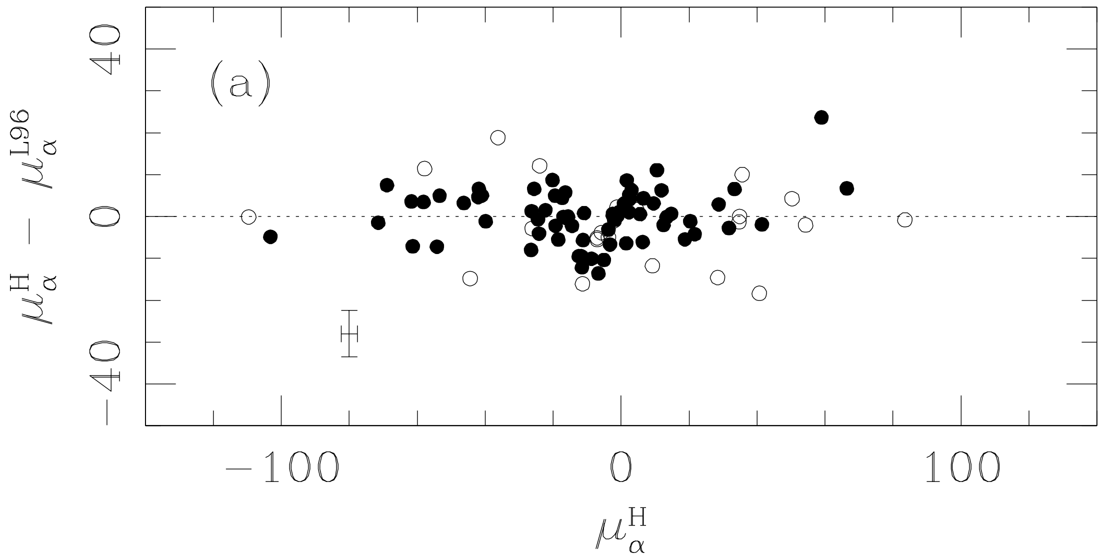
$$\mu_\alpha^H - \mu_\alpha^{L96} = -0.29 \pm 0.73, \quad \text{rms} = 7.04 \ \ \text{mas yr}^{-1},$$
$$\mu_\delta^H - \mu_\delta^{L96} = +0.71 \pm 0.80, \quad \text{rms} = 7.68 \ \ \text{mas yr}^{-1}.$$

Tsujimoto et al. (1998) reported finding a significant rotation between the Layden et al. (1996) and *Hipparcos* proper motion systems, with a total amplitude of $\sim$5 mas yr^{-1}. Popowski & Gould (1998b) disputed this result. In a study of non-variable stars, Platais et al. (1998) found no evidence for such rotation, but they did detect a significant, magnitude-dependent difference between the *Hipparcos* and Lick proper motions. However, the agreement between statistical parallax solutions using ground-based and *Hipparcos* proper motions, holding all other inputs fixed, shows that this difference is entirely negligible for the purposes of statistical parallax (PG98b).

Since the release of the *Hipparcos* data, five groups have performed statistical parallax solutions. Each group has employed a slightly different algorithm, and has adopted slightly different input data and assumptions. However, the set of stars employed and much of the data for them remains very similar from one study to the next, so it is not surprising that the results from all five groups are very similar. Nevertheless, the agreement provides reassurance that the general method is not susceptible to small variations in technique or input.

The investigation of Tsujimoto, Miyamoto, & Yoshii (1998) was among the first to be published. Their statistical parallax algorithm was similar to that of Hawley et al. (1986, also Murray 1983). Their data set consisted of proper motions from *Hipparcos*, and radial velocities, apparent magnitudes, interstellar extinction, and [Fe/H] values from Layden et al. (1996). They obtained $M_V(RR) = 0.69 \pm 0.10$ mag for a sample of 99 halo stars with $\langle[\text{Fe/H}]\rangle = -1.58$ dex.[5] Gould & Popowski (1998) have questioned the error value on this result, since it is smaller than their analytically-derived minimum error value (see below).

[5]I refer the reader to the individual papers for the mean velocities (U, V, W) and velocity dispersions $(\sigma_U, \sigma_V, \sigma_W)$ resulting from the solutions. The $M_V(RR)$ results for each study are summarized in Table 1.



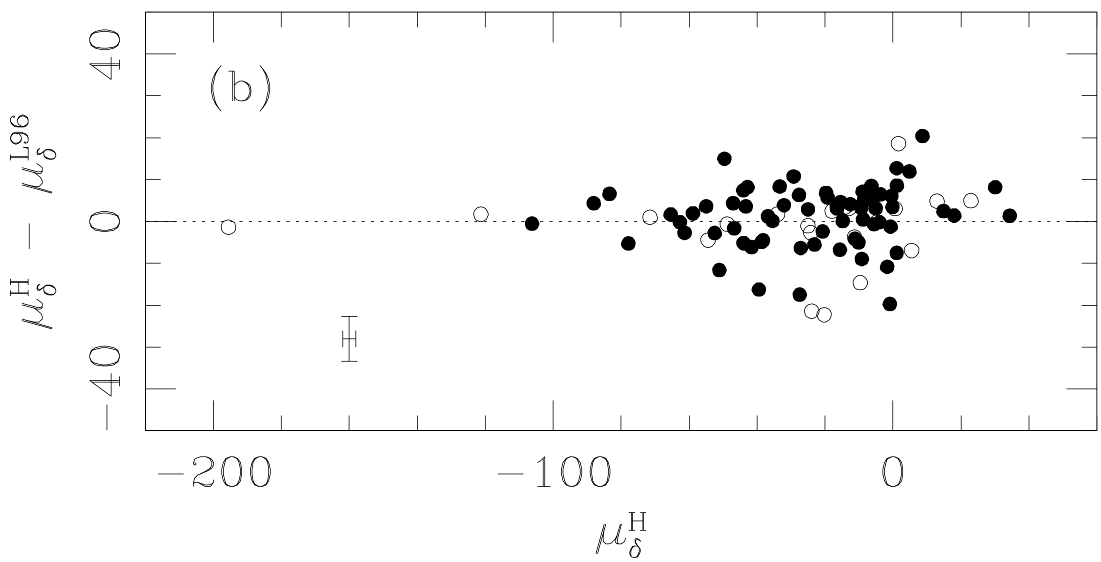


Fig. 1.— Comparison of *Hipparcos* proper motions with those used by Layden et al. (1996) in (a) Right Ascension and (b) Declination. Units are mas yr^{-1}. Ground-based data are from NPM (•) and WMJ (∘). Error bars indicate the mean errors.

The solutions of Fernley et al. (1998) present several improvements upon the Tsujimoto et al. (1998) work. First, Fernley et al. improved the reddening estimates of some low-latitude stars using observed $(V-K)$ colors (for most of the stars, they took reddening values from the maps of Burstein & Heiles, 1982). They also recompiled the radial velocity and metallicity data from the original sources, most of which were employed in the Layden et al. (1996) compilation. Most importantly, they derived new apparent magnitudes for most of the stars from the *Hipparcos* photometry database. After rejecting a number of stars of questionable value, they used the Hawley et al. (1986) statistical parallax algorithm to obtain $M_V(RR) = 0.77 \pm 0.17$ mag for 84 halo RR Lyrae (defined as [Fe/H] < -1.3, $\langle$[Fe/H]$\rangle = -1.66$). They also obtained $M_V(RR) = 0.76 \pm 0.13$ mag for all 144 RR Lyrae in their sample, but they note that this involves a dynamically heterogeneous set of halo and thick disk stars, and they therefore prefer the halo-only solution.

Heck & Fernley (1998) provide some interesting comparisons between the results of two different statistical parallax algorithms. Using the data from Fernley et al. (1998), they compare the Fernley et al. statistical parallax results with those of the statistical parallax algorithm of Heck (1975). An interesting aspect of Heck's code is that it performs internal tests to ensure that the sample is appropriately homogeneous, and rejects from the solution any stars which deviate greatly from the overall parameter distributions. Using the entire 144 star data set, they obtained $M_V(RR) = 0.78 \pm 0.13$ (one star rejected), and using only the 84 halo stars, they obtained $M_V(RR) = 0.81 \pm 0.15$ (no rejections). Contrary to some previous criticisms, the two methods produce nearly identical results in a controlled, real-data comparison.

Luri et al. (1998) also directly employed the Fernley et al. (1998) data set with one exception; they adopted the Arenou et al. (1992) interstellar absorption model. They input the entire dataset into their statistical parallax algorithm (Luri et al. 1996, the "LM-method") which, in addition to rejecting outliers, identifies, segregates, and produces solutions for any self-consistent groupings it finds in parameter space. They find a grouping of 113 stars with $\langle$[Fe/H]$\rangle = -1.51$ and $M_V(RR) = 0.65 \pm 0.23$ which they associate with the halo, and a second grouping of 18 stars ($\langle$[Fe/H]$\rangle = -0.45$, $M_V(RR) = 0.12 \pm 0.49$) which they associate with the disk. Luri et al. attribute the difference between their results and those of Fernley et al. (1998) to the differences in star assignments to disk or halo groupings, but they do not mention whether systematic differences between the adopted reddening systems contribute as well. The M_V errors quoted by Luri et al. are based on the scatter of multiple Monte Carlo simulations, and thus include the shot noise associated with drawing a finite sample out of a smooth distribution, a factor which the error estimates of other studies do not include (though the Monte Carlo simulations of Layden et al. (1996) and PG98a indicate that their internal error estimates are reliable, perhaps even over-estimated). The larger errors quoted by Luri et al. (1998) may also reflect the larger number of variables for which they solve (e.g., disk scale height Z_0, apparent magnitude limit V_c, etc). Luri and his colleagues (private communication) are performing simulations to test for any bias incurred by parameterizing the distribution of halo RR Lyrae stars with an exponential disk.

The fifth study of RR Lyrae stars appears in the three paper series by Popowski & Gould. A very interesting result of their first paper (PG98a) is their analytic expression for the relative error in the distance scaling parameter, from which $M_V(RR)$ is computed. They show that for a population of stars with given velocity dispersions and bulk motions, and which have observational errors smaller than the velocity dispersions, the relative error in the distance scaling parameter is proportional to $N^{-1/2}$, where N is the number of stars in the sample. Thus, in the case of the halo RR Lyrae sample, where observational errors in the radial and tangential velocities are typically 20–30 km s^{-1} compared with velocity dispersions $\sim$100 km s^{-1}, improving the quality of the proper motions produces little effect. The only way to improve the results is to include more stars. This explains why the errors in $M_V(RR)$ quoted by the post-*Hipparcos* statistical parallax studies of Fernley et al. (1998) and Heck & Fernley (1998) *increased* relative to that quoted by Layden et al. (1996). Those studies used improved proper motions, but contained fewer stars.

In their second and third papers, PG98b and Gould & Popowski (1998, hereafter GP98), strove to increase the number of stars in the solution by including stars from Layden et al. (1996) with ground-based proper motions. They searched for systematic differences between the *Hipparcos* and ground-based proper motions, and rejected all stars with questionable proper motions. They also determined that the radial velocity observations and their estimated errors are not a source of significant systematic error to the statistical parallax solutions (PG98b). In GP98, they used the *Hipparcos* photometry of Fernley et al. (1998) to show that the apparent magnitude system adopted by Layden et al. (1996) is too bright by 0.06 mag, and they created a new, self-consistent set of photometry. Finally, they adopted the reddening maps of Schlegel, Finkbeiner & Davis (1998), which are derived from direct measurements of far infrared dust emission, rather than the indirect HI maps of Burstein & Heiles (1982). They ultimately obtained $M_V(RR) = 0.77 \pm 0.13$ mag for 147 stars with $\langle[\mathrm{Fe/H}]\rangle = -1.60$ mag. This result includes a correction for Malmquist bias (0.03 mag), and several other biases at the 0.01 mag level (see GP98, PG98a).

After noting the precision limitations placed on the RR Lyrae statistical parallax solutions by the limited number of RR Lyraes with observed proper motions, PG98b developed a hybrid statistical parallax method whereby large samples of halo stars which are not RR Lyrae stars and which do not have proper motions can contribute to the determination of $M_V(RR)$. The radial velocities of all the stars are used to determine the halo velocity ellipsoid. This distance-independent ellipsoid is then matched to the distance-dependent ellipsoid defined by the RR Lyrae proper motions via maximum likelihood adjustment of the distance scale parameter. They give plausible arguments why it is safe to assume that the non-variables and RR Lyrae stars sample the same kinematic stellar population. As the number of stars contributing radial velocities becomes large, the error in the distance scale parameter approaches $3^{-1/2}$ times the corresponding error in the standard statistical parallax solution. GB98 apply this method to 716 non-variables and 87 RR Lyrae with [Fe/H] < -1.5 ($\langle$[Fe/H]$\rangle = -1.81$) and find $M_V(RR) = 0.82 \pm 0.13$, in agreement with their standard statistical parallax result. Their error estimate includes a contribution due to possible differences in thick disk contamination between the two samples. They combine the results of their standard and hybrid solutions, accounting for the correlation between them, to obtain $M_V(RR) = 0.80 \pm 0.11$ mag at $\langle$[Fe/H]$\rangle = -1.71$ dex.

Several of the groups (Layden et al. (1996), Luri et al. 1996, PG98a,b) used Monte Carlo simulations to test their algorithms for biases. This generally involves drawing a large number of simulated data sets from distribution functions approximating those of the observed stars, then performing a statistical parallax solution on each data set, and comparing the derived luminosity and kinematic parameters to those of the parent distribution functions. For each parameter, the dispersion of the individual tests about the mean is an estimate of the error inherent in the solution (Luri et al. 1996 adopt this as their error value, while Layden et al. (1996) and PG98a,b use it to confirm their error values). In general, all the Monte Carlo tests show that the statistical parallax solutions return their input values to within the quoted, realistic errors. Among other things, PG98a tested their algorithm for sensitivity to (a) the assumed size of the intrinsic scatter in $M_V(RR)$ and the associated Malmquist bias, (b) the distribution of observed stars on the sky, and (c) deviations in the shapes of the parent velocity distributions from Gaussian. None of the resulting biases in $M_V(RR)$ were larger than 0.03 mag, and they tended to act in opposite senses to roughly offset each other. Luri et al. (1998) have reserved the details of their Monte Carlo tests for a forthcoming paper. Clearly, the RR Lyrae statistical parallax results have been tested rigorously for sources of internal bias, and none are found which compromise the results.

4. Application to Cepheid Variables

Upon searching the usual electronic abstract and preprint sources, I was surprised to find only one paper which applies *Hipparcos* data to a statistical parallax study of Cepheid variables.

Luri et al. (1998) used astrometric, photometric, and period data from the *Hipparcos* Catalogue (ESA 1997) and radial velocities from the *Hipparcos* Input Catalogue (Turon et al. 1992) to compile a sample of 219 classical Cepheids with all known overtone Cepheids eliminated. They adopted a period-luminosity relation $M_V(Cep) = \alpha + \beta \log P$, and produced two solutions, one with $\beta = -2.81$ (adopted from Cepheids in the Large Magellanic Cloud), and one with β as a free parameter. In the first case, they found $\alpha = -1.05 \pm 0.17$ mag, and in the second case, $\alpha = -1.73$ and $\beta = -2.12$ mag (with an error in M_V of $0.20 + 0.08 \log P$). Both cases produced results significantly fainter than recent trigonometric parallax calibrations, the former being 0.38 mag fainter than the Feast & Catchpole (1997) result. Using this Cepheid calibration, Luri et al. (1998) determined the distance modulus to the LMC to be 18.25 ± 0.18 mag, in excellent agreement with the RR Lyrae value of 18.20 ± 0.14 (computed from PG98a and GP98).

Since most of the Cepheids lie near the Galactic plane, the treatment of reddening is especially important for the Cepheid calibrations. Luri et al. (1998) used the Arenou et al. (1992) three-dimensional reddening maps, which have a rather coarse sampling on the sky. More accurate reddenings for individual Cepheids are obtainable from the optical or near-IR colors of the Cepheids in question. Luri et al. performed an alternate solution using the BVI-derived reddenings from Feast & Catchpole (1997), and obtained results similar to the ones using the Arenou reddenings: $\alpha = -1.74$ and $\beta = -2.04$ mag. The agreement of the two reddening treatments bolsters confidence in the overall result.

Luri, Gómez, Beaulieu, & Goupil (1999) are continuing their work on Cepheids. Noting the shallow period-luminosity slope β derived in their previous work, they divided the Cepheid sample into stars with periods greater and less than 10 days. The long-period group produced a slope in good agreement with the LMC value, while the short-period group gave a very shallow slope, $\beta \approx -1.4$ mag. They suspect that the short-period group is contaminated by undetected overtone Cepheids. They then applied to the total 219 star sample a specialized version of the LM-method which imposes the existence of two PL relations, one for the fundamental pulsators and a second for the overtone pulsators. As a first approximation, the two sequences were supposed to be parallel and separated by $P_1/P_0 = 0.72$. Their provisional results are $\beta = -2.6$, and $\alpha = -1.04$ mag for the fundamental pulsators. Thus their derived slope is much closer to the LMC value than before, while the overall relation still favors a faint absolute magnitude ($\sim$0.5 mag fainter than Feast & Catchpole (1997) at the median Cepheid period).

These results may be compared with the pre-*Hipparcos* results of Wilson et al. (1991) for 90 classical Cepheids, $\alpha = -1.21 \pm 0.33$ assuming the LMC slope of $\beta = -2.81$. This zero-point lies midway between the results of Luri et al. (1998) and of Feast & Catchpole (1997).

It would certainly be useful for the other statistical parallax groups (§3) to perform analyses for the Cepheids, employing their distinct algorithms and assumptions for reddening, etc. Moreover, rigorous Monte Carlo tests must be published before the Cepheid statistical parallax results can be trusted as securely as the RR Lyrae results. In particular, does the inhomogeneous distribution of Cepheids on the sky produce a bias in the statistical parallax solution? Also, with the large intrinsic magnitude spread of Cepheids, can an accurate Malmquist bias correction be obtained? For now, the Luri et al. (1998) results provide hope that the RR Lyrae and Cepheid absolute magnitude scales can be reconciled.

5. Application to Other Stellar Types

For main sequence stars, there are usually enough objects near the Sun of a well-defined spectral type or color to make trigonometric parallaxes the preferred method for determining absolute magnitudes. However, the samples become small for very early type stars, for some highly evolved stars, and for chemically peculiar stars. The LM-method of statistical parallax (Luri et al. 1996) has been applied to a number of these stellar classes. While the results are interesting for determining the masses and evolutionary status of the stars, they are less applicable to the cosmic distance scale, and so I shall merely highlight some of the findings.

Gómez et al. (1997) applied the LM-method to large samples of *Hipparcos* stars which span a wide range in spectral types and luminosity classes. Since the LM-method produces M_V estimates for individual stars, in addition to the mean M_V value for the sample, they were able to place the individual stars in the color vs. absolute magnitude diagram. They performed solutions for each of the five luminosity classes, I–V, employing stars with spectral types ranging from B to K. The color-magnitude diagram for each class shows a large scatter in M_V at a given color, and stars of a given luminosity class do not define unique regions in the color-magnitude plane. The authors thus provided a striking reminder that spectroscopic parallaxes have a very low intrinsic accuracy.

Gómez et al. (1998) performed similar analyses on sets of chemically peculiar B and A stars including He-rich and He-weak stars (spectral types B2–B8), Silicon stars (B7–A2 types), and Am stars (A0–F0 types). Again, the LM-method was used to place individual stars in the HR diagram. Each group of stars was seen to populate the full range of main sequence absolute magnitudes at a given effective temperature, that is, from the Zero Age Main Sequence to hydrogen exhaustion in the core. Intrinsic dispersions in M_{bol} were 0.5–0.8 mag. Both the absolute magnitudes and kinematics appear to be in agreement with normal main sequence stars of comparable spectral type.

Mennessier et al. (1997) used the group identification and separation feature of the LM-method to separate a sample of 297 Barium stars into five groups. A halo group consisting of subdwarfs and giants separated out because of its extreme kinematics. Four groups with disk kinematics separated out by location in the color vs. absolute magnitude diagram. These groups comprised dwarf, red giant, supergiant, and red-clump giant stars, respectively. The authors demonstrated the heterogeneous nature of the Barium stars and interpreted the five groups in the context of current pictures of Barium star production through mass donation from an evolved companion.

6. Conclusions and the Future

Most of the statistical parallax work which employs *Hipparcos* data has focused on RR Lyrae variables, and so the main conclusions of this paper concern those stars. I have described how, with the advent of high precision *Hipparcos* proper motions and uniform *Hipparcos* photometry, several groups have greatly improved the database used in RR Lyrae statistical parallax solutions. Furthermore, Popowski & Gould have used these data to search for and remove systematic errors from pre-*Hipparcos*, ground-based data, and thus enter it into the statistical parallax solutions on a fair footing (PG98b, GP98). This is important because, as those authors have shown, the uncertainty in a statistical parallax solution scales as $N^{-1/2}$, where N is the number of stars in the solution. Finally, the solutions, whose results depend on a maximum likelihood analysis employing a rather complicated model of Galactic dynamics, have been performed by several groups using independent algorithms. These groups obtain very similar results, indicating that implementation problems or specific assumptions such as reddening corrections are not producing spurious results. Several of the groups have performed detailed Monte Carlo tests to search for biases produced by the shortcomings of the statistical parallax model, non-uniform distribution of the stars on the sky, etc. The biases are always much smaller than the quoted uncertainties (typically 0.03 mag or less), and corrections for them usually can be applied. I therefore argue that the statistical parallax solutions represent a very mature, well-tested result which can not be dismissed lightly.

In Table 1 I have summarized the results of the post-*Hipparcos* RR Lyrae statistical parallax solutions, along with the pre-*Hipparcos* results of Layden et al. (1996) for comparison. The columns contain the following quantities: (1) a reference to the study in question, (2) the number of halo stars employed (thick disk stars were excluded), (3) the mean metallicity of the halo sample, (4) the RR Lyrae absolute magnitude resulting from the solution and its error, and (5) that value normalized to [Fe/H] = –1.60 dex using $\Delta M_V/\Delta[\mathrm{Fe/H}] = 0.18$ mag dex^{-1} (Fernley et al. 1997). Considering the large sample size and attention to systematic errors given by the Gould & Popowski (1998) study, I adopt this as the preferred statistical parallax zero-point, $M_V(RR) = 0.77 \pm 0.13$ mag at $\langle[\mathrm{Fe/H}]\rangle = -1.60$ dex.

Table 1: Statistical Parallax Solutions for RR Lyrae Stars

Reference	N_{halo}	$\langle[\mathrm{Fe/H}]\rangle$	$M_V(RR)$	$M_V(RR)^a_{-1.6}$
Layden et al.	162	–1.61	0.71 ± 0.12	0.71
Tsujimoto et al.	99	–1.58	0.69 ± 0.10	0.69
Fernley et al.	84	–1.66	0.77 ± 0.17	0.78
Heck & Fernley	84	–1.66	0.81 ± 0.15	0.82
Luri et al.	113	–1.51	0.65 ± 0.23	0.63
Gould & Popowski[a]	147	–1.60	0.77 ± 0.13	0.77
Gould & Popowski[a]	87	–1.81	0.82 ± 0.13	0.86

[a] see text.

Thus, the statistical parallax results for field RR Lyrae stars near the Sun remain in conflict at the 2σ level with several other determinations of $M_V(RR)$, several of which employ RR Lyrae stars in globular clusters (see other chapters in this volume). GP98 suggest some possible causes. First, the stars in the statistical parallax sample may represent a 1-in-20 statistical fluctuation away from the underlying population of halo stars which results in determining $M_V(RR)$ too faint. There is no way of testing this short of greatly increasing the number of RR Lyrae stars in the sample. Second, there may be an intrinsic difference between the magnitudes of field and cluster RR Lyrae. However, Baade-Wesselink luminosities of field and cluster RR Lyrae provide marginal evidence against this scenario (e.g., Storm et al. 1994), and GP98 note that (a) field and cluster RR Lyrae in the LMC have nearly identical magnitudes, and (b) the period-temperature diagrams for Galactic field and cluster RR Lyrae stars are similar at similar metallicities. More work is required to determine whether this is the cause of the discrepancy. Third, PG98 outline how differences in the metallicity scales between local subdwarfs and cluster giants can bias the results of main sequence fitting techniques toward brighter values of $M_V(RR)$. One thing is clear, however. *Systematic* errors in the statistical parallax results for $M_V(RR)$ are *not* the cause of this conflict.

In addition to the RR Lyrae analyses, I have briefly reviewed several statistical parallax studies of chemically peculiar stars and other stellar classes. I have also discussed the one statistical parallax study of Cepheid variables which has, as of this date, employed *Hipparcos* data. Luri et al. (1998) obtained a period-luminosity relation with a zero-point 0.38 mag fainter than that determined by Feast & Catchpole (1997) from their statistical treatment of Cepheid trigonometric parallaxes. The Luri et al. calibration results in an Large Magellanic Cloud distance modulus of 18.25 ± 0.18 mag, in excellent agreement with the RR Lyrae statistical parallax results, $\mu_{LMC} = 18.20 \pm 0.14$ mag (PG98a, GP98). While this agreement is heartening, more work needs to be done on the Cepheids, in particular more Monte Carlo tests for statistical bias, before their statistical parallax absolute magnitudes are as rigorously tested as those of the RR Lyrae stars.

Despite the promising agreement between the RR Lyrae and Cepheid absolute magnitude scales as determined by statistical parallax, these results remain in conflict with many recent calibrations of the Cepheid period-luminosity relation, including the trigonometric parallax determinations of Feast & Catchpole (1997) and others. It should be noted that the 26 *best* Cepheids from the *Hipparcos* Catalogue have a mean relative error of $\sigma_\pi/\pi = 0.6$, so a careful statistical treatment is required to obtain an accurate result, and Luri et al. (1998) have criticized the treatment used by Feast & Catchpole (1997). Still, the statistical parallax results also conflict with other Cepheid calibrations such as main sequence fits to open clusters containing Cepheids. What might be the cause? First, PG98a have shown that the observed magnitudes and reddenings of field RR Lyrae stars in the LMC can be improved. However, it seems unlikely that this alone will reconcile the $\sim$0.3 mag difference in the RR Lyrae and Cepheid distance moduli. Second, it is sometimes suggested that the absolute magnitudes of field RR Lyrae stars in the LMC differ from those near the Sun, so using local calibrations to obtain μ_{LMC} is invalid. While the $M_V(RR)-$[Fe/H] relation is fairly well established, it is possible that the relative abundances of light elements (e.g., He, C, N, O, etc.) differ. These parameters are known to affect the luminosity of the horizontal branch, but direct measurements of them in halo LMC stars remains difficult. However, various studies of the Galactic halo suggest it formed through the accretion of many independently-evolving dwarf galaxies, akin to the early LMC, so perhaps the chemical compositions of the LMC and Galactic halos are not so dissimilar as some suggest. The same can not be said for the Cepheids. The metallicity sensitivity of the period-luminosity relation remains controversial, and the star formation histories of the LMC and Galactic disks are rather different. Thus there seems to be more uncertainty in the Galaxy–LMC connection for Cepheids than there is for RR Lyrae stars. In summary, there remain many details which must be worked out before the RR Lyrae and Cepheid distance scales can be fully reconciled.

What is the future of statistical parallax analyses? Luri and collaborators are extending their investigations using the LM-method to other classes of stars (Luri, private communication). I have already outlined the additional work needed on Cepheid variables. Even at their mature state, there is room for improvement in the RR Lyrae analyses. A meager improvement can be made by obtaining improved photometry for the $\sim$40 stars noted by GP98 to have sub-standard apparent magnitude or reddening estimates. A larger improvement will be seen when new ground-based proper motions become available for the fainter Southern Hemisphere stars (e.g., van Altena et al. 1990). Even in the North, the Lick Northern Proper Motion program (Klemola et al. 1993) has determined proper motions for large numbers of fainter RR Lyrae which only require radial velocities, abundances, and apparent magnitudes to be included in a statistical parallax solution. Combining these steps should increase the usable sample by 100 or more stars. In years to come, the *SIM* and *GAIA* satellites will provide superior quality proper motions for stars fainter than were observable with *Hipparcos*. Still other approaches are possible. PG98b developed the "poor-man's route" to statistical parallaxes, whereby a large sample of stars with radial velocities alone is used to determine the velocity ellipsoid (see §3). Using thousands of radial velocity stars, improvements could be made to the halo RR Lyrae statistical parallax solutions without obtaining any new proper motions. Radial velocity surveys of halo stars are currently underway which will yield the required sample. Finally, stable horizontal branch stars just blueward of the RR Lyrae instability strip could be included in the RR Lyrae solutions. Photometry and radial velocities are already available for hundreds of such stars (e.g., Beers et al. 1996), so they should be included in all proper motion input catalogues. Without doubt, the statistical parallax method will continue to make important contributions to the determination of the cosmic distance scale.

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References

Arenou, F., Grenon, M., & Gómez, A.E. 1992, A&A, 258, 104

Beers, T.C., Wilhelm, R. Doinidis, S.P. & Mattson, C.J. 1996, ApJS, 103, 433

Burstein, D. & Heiles, C. 1982, AJ, 87, 1165

ESA 1997, "The *Hipparcos* Catalogue", ESA SP-1200

Feast, M.W. & Catchpole, R.M. 1997, MNRAS, 286, L1

Fernley, J., Carney, B., Skillen, I., Cacciari, C., & Janes, K. 1997, MNRAS, 293, 61

Fernley, J., Barnes, T.G., Skillen, I., Hawley, S.L., Hanley, C.J., Evans, D.W., Solano, E., & Garrido, R. 1998, A&A, 330, 515

Gómez, A.E., Luri, X., Mennessier, M.O., Torra, J., & Figueras, F. 1997, in "Hipparcos Venice '97", ESA SP-402

Gómez, A.E., Luri, X., Sabas, V., Grenier, S., Figueras, F., North, P., Torra, J., & Mennessier, M.O. 1998, Contrib. Astron. Obs. Skalnaté Pleso, 27, 171; also astro-ph/9805017

Gould, A. & Popowski, P. 1998, ApJ in press; also astro-ph/9805176 (GP98)

Hawley, S.L., Jefferys, W.H., Barnes, T.G., & Wan, L. 1986, ApJ, 302, 626

Heck, A. 1975, Ph.D. Thesis, Univ. Liège

Heck, A. & Fernley, J. 1998, A&A, 332, 875

Klemola, A.R., Hanson, R.B., & Jones, B.F. 1993, Lick Northern Proper Motion Program: NPM1 Catalog, National Space Science Data Center – Astronomical Data Center Catalog No. A1199 (NPM)

Layden, A.C., Hanson, R.B, Hawley, S.L., Klemola, A.R. & Hanley, C.J. 1996, AJ, 112, 2110

Luri, X., Gómez, A.E., Torra, J., Figueras, F., & Mennessier, M.O. 1998, A&A, 335, 81

Luri, X., Gómez, A.E., Beaulieu, J.P., & Goupil, M.J. 1999, in preparation

Luri, X., Mennessier, M.O., Torra, J., & Figueras, F. 1996, A&AS, 117, 405

Mennessier, M.O., Luri, X., Figueras, F., Gómez, A.E., Grenier, S., Torra, J., & North, P. 1997, A&A, 326, 722

Murray, C.A. 1983, “Vectorial Astrometry”, (Adam Hilger Ltd: Bristol), 297

Platais, I., Kozhurina-Platais, V., Girard, T.M., van Altena, W.F., López, C.E., Hanson, R.B., Klemola, A.R., Jones, B.F., MacGillivray, H.T., Yentis, D.J., Kovalevsky, J., & Lindegren, L. 1998, A&A, 331, 1119

Popowski, P. & Gould, A. 1998, ApJ, in press; also astro-ph/9703140 (PG98a)

Popowski, P. & Gould, A. 1998, ApJ, in press; also astro-ph/9802168 (PG98b)

Schlegel, D.G., Finkbeiner, D.P. & Davis, M. 1998, ApJ, 500, 525

Storm, J., Carney, B.W., & Latham, D.W. 1994, A&A, 290, 443

Turon, C., Crézé, M, Egret, D., et al. 1992, “The *Hipparcos* Input Catalog”, ESA, SP-1136

Tsujimoto, T., Miyamoto, M., & Yoshii, Y. 1998, ApJ, 492, L79

van Altena, W.F., Girard, T., López, C.E., López, J.A., & Molina, E. 1990, Proc. IAU Symp. 141, edited by J. Lieske & V. Abalakin, (Kluwer: Dordrecht), 419

Wilson, T.D., Barnes, T.G., Hawley, S.L., & Jefferys, W.H. 1991, ApJ, 378, 708