Photometry of the Globular Cluster M54 and the Sagittarius Dwarf Galaxy: the Age-Metallicity Relation

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ABSTRACT

We present deep VI-band photometry of the globular cluster M54, a nearby field in the Sagittarius dwarf galaxy, and a control field. The color-magnitude diagrams reach well below the oldest main sequence turn-offs, thus enabling an analysis of the galaxy's age-metallicity relation with unprecedented clarity. We also study the variable stars in the direction of M54. From 67 RR Lyrae variables, we confirm and improve on our previous estimates of the cluster horizontal branch magnitude, foreground reddening, and horizontal branch morphology. These values are used in determining the ages of M54 and the Sagittarius field populations. We confirm our previous result that M54 is the same age as Galactic globular clusters of similar metallicity. We also derive ages on a self-consistent scale for the other three globular clusters in Sagittarius. We find strong evidence for multiple episodes of star formation (or continuous star formation with a variable rate) in the field of Sagittarius. We characterize the principal episodes with the ages 11, 5, and 0.5 through 3 Gyr, and with $[Fe/H]$ values of –1.3, –0.7, and –0.4, respectively. On this scale, M54 has an age of 15 Gyr. Surprisingly, the age-metallicity relation we have derived for the galaxy as a whole is described quite well by a closed box chemical evolution model. We also find that the populations associated with the Sgr field are clumped spatially around M54, and we consider several explanations for this phenomenon. We again speculate that Sagittarius is a nucleated dwarf elliptical galaxy with M54 as its nucleus.

Subject headings: galaxies: stellar content — galaxies: star clusters — galaxies: individual (Sagittarius) — globular clusters: individual (M54) — stars: variables: general

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1. Introduction

The study of the star formation history of dwarf galaxies is interesting for several reasons. First, dwarf galaxies are among the most frequently-occurring galaxies in the universe. Second, they may account for much of the blue galaxy population seen at intermediate redshift (e.g., Driver et al. 1995). Finally, theories of structure formation in the cold dark matter paradigm involve building luminous galaxies from numerous smaller fragments; observational evidence suggests that the dwarf spheroidal galaxies we observe today may represent such fragments which were not assimilated into larger galaxies (e.g., Searle & Zinn 1978).

The study of dwarf galaxies is hampered by their large distances. In the early 1990's, only six dwarf spheroidals were known to lie within 150 kpc of the Galaxy, the nearest being Ursa Minor at 66 kpc (Mateo 1998). Color-magnitude diagram studies of their stellar populations were largely limited to the horizontal branch and brighter before the repaired *Hubble Space Telescope* was able to reach their main sequence turn-off levels.

It was therefore very exciting when Ibata et al. (1994) discovered a gas-poor galaxy lying behind the bulge of the Milky Way at a distance of only ~25 kpc. In addition to mapping out the field population of this galaxy, now known as the Sagittarius Dwarf Galaxy (hereafter "Sgr"), they suggested that four previously-known globular clusters (M54, Arp 2, Ter 7, & Ter 8) are members of this galaxy. Sgr thus provides an unprecedented opportunity to study the star formation history of a gas-poor dwarf. The main complication is confusion in the color-magnitude diagrams from the vast number of foreground bulge stars which served to veil our nearest neighbor for so long.

Ibata et al. (1997) reviewed much of the early work on Sgr. Several relevant points include the radial velocity study of Da Costa & Armandroff (1995) which confirmed the membership of the globular cluster M54 (NGC 6715), and the first CCD color-magnitude diagram (CMD) of this cluster by Sarajedini & Layden (1995, hereafter referred to as SL95). They found M54 to be metal poor ($[Fe/H] = -1.79$ dex) and to have a blue horizontal branch typical of old Galactic globular clusters. The other globular clusters have been studied by Buonanno et al. (1995a, Arp 2), Buonanno et al. (1995b, Ter 7), and Montegriffo et al. (1998, Ter 8).

Mateo et al. (1995a, hereafter referred to as MUSKKK) were the first to obtain main sequence turn-off photometry of the Sgr field population. They characterized it with a metallicity of $[Fe/H] = -1.1$ dex and an age of ~10 Gyr, and also noted the presence of a small population of younger-aged stars. Mateo et al. (1996) and Fahlman et al. (1996) found similar ages and metallicities for the dominant Sgr field population at a location far distant from the dense, central regions of Sgr. Marconi et al. (1998) found their Sgr field photometry to closely resemble the CMD of the young globular Ter 7. Layden & Sarajedini (1997) found Sgr to be at least 3 Gyr younger than typical Galactic globular clusters.

More recently, Bellazzini et al. (1999a, 1999b) have obtained deeper photometry of the Sgr field populations at three locations in the galaxy. They found the bulk of the Sgr field stars to have an age ~4 Gyr younger than those of the Galactic globular clusters. They also suggested the young population discovered by MUSKKK has an age of 1–4 Gyr and $[Fe/H] \approx -0.7$ dex. They raised the possibility that the dominant Sgr main sequence turn-off (MSTO) region, previously considered to be a single population, may in fact be composed of two or more populations with different ages and metallicities. However, the limited photometric depth and large errors near the MSTO in their survey prevented them from quantifying the ages of these populations. They discussed their results in the context of the global star formation history of Sgr.

The globular cluster M54 must play an important role in the star formation history of Sgr. It lies in the highest density region of Sgr (Ibata et al. 1997), it is one of the most luminous globular clusters known (SL95), and as we shall show, it marks one of the earliest epochs of star formation in Sgr. SL95 suggested that M54 may be akin to the nuclear star clusters seen in nucleated dwarf elliptical galaxies.

Marconi et al. (1998) presented photometry of M54 which just reached the MSTO region. From it, they estimated that the age of M54 is similar to that of the Galactic globular clusters. Deeper photometry by Layden & Sarajedini (1997, hereafter referred to as LS97) provided more convincing evidence that the age of M54 is typical of Galactic globular clusters with the same metallicity.

In this paper, we discuss in detail our deep photometric observations of M54 which were presented in preliminary form by LS97. We also present observations of a nearby field in Sgr, and of a control field selected to represent the contaminating foreground populations. In §2 we describe our observations, and in §3 we detail the photometric reduction techniques. We describe the resulting color-magnitude diagrams in §4 and perform statistical subtractions of the contaminating populations. We digress in §5 to study the variable stars in our M54 field, and derive from them in §6 several important cluster parameters, including the mean apparent magnitude of the cluster RR Lyrae stars, the foreground reddening, and the horizontal branch morphology of M54. In §7 we estimate the ages of M54 and the other globulars in Sgr. In §8 we present clear evidence for temporally-extended star formation in the Sgr field, and characterize the ages of these populations. We consider the age-metallicity relation of the Sgr galaxy, including its globular clusters, in §9, while in §10 we demonstrate that the Sgr field populations are clumped around M54. We summarize our results and consider their larger meaning in §11.

2. Observations

Observations of the bright sequences ($M_V \lesssim +2.0$) of M54 were obtained with the 0.9-m telescope at CTIO, as reported in SL95. These included photometry obtained with the Tek#1 1024×1024 pixel CCD (Field of View (FOV) $\approx$ 6.8 arcmin) on a field roughly centered on M54, and on another field 12 arcmin North of M54 (hereafter referred to as the Sgr field). Separate datasets were presented for these fields from observations made with the Tek#3 2048×2048 pixel CCD (FOV $\approx$ 13.5 arcmin). The quality of the latter sets was compromised by small, spatially-dependent variations in throughput which varied with time. We hereafter disregard the SL95 Tek 2048 photometry and when referring to the SL95 work, and consider only the Tek 1024 data.

In an effort to obtain photometry to the MSTO, we secured deep images of M54 using the CTIO 4.0-m telescope at prime focus during the nights of 1995 June 28 and 29. We employed the Tek#4 2048×2048 pixel CCD, which provided a scale of 0.43 arcsec pixel^{-1}. The filters were matched to the CCD response to closely approximate the standard Johnson V and Kron-Cousins I passbands.

Each "pointing" consisted of one 600 sec V exposure and two 300 sec I exposures. Nine such pointings, centered on M54, were obtained during the course of the run. The fwhm seeing ranged from 1.0 to 1.5 arcsec (median = 1.2 arcsec). The equatorial and galactic coordinates of the field center are shown in Table 1. Eight pointings of the Sgr field were taken, with seeing ranging from 1.2 to 2.1 arcsec (median = 1.4 arcsec). Four pointings near the MUSKKK bulge control field were also obtained, with seeing ranging from 1.1 to 1.3 arcsec (median = 1.1 arcsec).

Short exposures of M54 were obtained in order to ensure a good photometric transformation between the SL95 and 4.0-m data, and to provide further information on the bright variable stars in the cluster. Five paired frames in V and I were obtained during the run, along with six additional, unpaired V frames. Exposure times were typically 30 sec in V and 25 sec in I.

The frames were overscan corrected, bias subtracted, and trimmed using the IRAF[3] task QUADPROC. Twilight sky flats were used to flat field the data.

3. Photometry

All the photometry described in this paper has been computed using the DoPHOT photometry program (Schechter et al. 1993). A version employing a 2-dimensionally variable point spread function enabled us to get good results despite the presence of significant radial variation in the width, shape, and orientation of the stellar images. Small variations not accounted for by the variable profile fitting function appeared as a 2-dimensionally variable zero-point offset in the photometry, and must be corrected for, either through a 2-dimensionally variable aperture correction or through other means as described below.

3.1. General Procedure

The high stellar densities in the Sgr field, and particularly in the M54 field, present a distinct observational challenge. Preliminary tests showed that while PSF fitting routines employing a small fitting radius (~fwhm) produced high quality instrumental magnitudes, the crowding of faint neighbor stars prevented us from obtaining accurate aperture corrections in our deep images.

Fortunately, in the M54 and Sgr fields, there exists VI CCD photometry of the bright stellar sequences. Crowding of the bright aperture correction stars on

[3]IRAF is written and distributed by NOAO.

these frames is less severe, so accurate aperture corrections were obtained.

We therefore transformed the instrumental DoPHOT photometry (v, i) directly onto the existing photometry (V, I) using the large number of stars in common. This transformation took the form

$$v - V = c_0 + c_1(V-I) + c_2X + c_3X^2 + c_4X^3 + c_5XY +$$
$$c_6X^2Y + c_7XY^2 + c_8Y^3 + c_9Y^2 + c_{10}Y$$

and

$$i - I = d_0 + d_1(V-I) + d_2X + d_3X^2 + d_4X^3 + d_5XY +$$
$$d_6X^2Y + d_7XY^2 + d_8Y^3 + d_9Y^2 + d_{10}Y,$$

where X and Y are the two spatial coordinates, and c_i and d_i are the i-th fitted coefficients. The details of the fits are described in the following subsections.

Note that the M54 photometry of Marconi et al. (1998) is in good agreement with that of SL95. This strengthens our confidence in the results of the photometric transformation procedure.

3.2. The Bright Sequences of M54 Revisited

We obtained five VI frame pairs with short exposure times centered on M54. Here, we use these images to extend slightly the areal coverage of the bright-sequence photometry relative to that presented in SL95. Not only does this provide us with more stars in the bright sequences, it is an important link for photometrically transforming the deeper MSTO photometry of M54, since much of the inner region sampled by SL95 is saturated or severely crowded in the deeper frames.

For each frame, we trimmed off the edges to remove the worst effects of the image variations, leaving a 11.1 arcmin FOV centered on the cluster. We then computed v and i instrumental stellar magnitudes using the DoPHOT program. For each frame pair, we matched the v and i photometry of the spatially coincident stellar images, thus creating five lists of instrumental photometry. Next, we performed (X, Y) coordinate transformations to put the coordinate frame of each photometry list onto a common system. We then used the equations shown in §3.1 to transform the instrumental magnitudes onto the standard VI magnitude system of SL95 (their Table 1). For each frame, the RMS scatter in the transformation was typically <0.030 mag, and the transformation always employed >800 stars covering $0.0 \lesssim V - I \lesssim 2.8$ mag and spatially spanning the image. Since the area of the 4.0-m frames is larger than that of the SL95 Tek 1024 data, we had to extrapolate the photometric transformation into the regions not covered by the SL95 photometry. However, the (X, Y) variations were small and well-behaved, so the extrapolation should not incur large errors.

We then compiled the photometry of spatially coincident stars from the five photometry lists, keeping only those stars which were detected in at least three of the five lists. For each star, we computed the weighted mean V and I magnitudes (the individual magnitudes were weighted by their DoPHOT errors). We also computed the standard error of the mean of the 3–5 V or I observations, which we treat as the error in the V or I magnitude. Table 2 presents, star by star, our final bright-sequence photometry set. The columns contain (1) an identification number, (2) the X and (3) Y coordinates in the system defined by our Figure 1, (4) the weighted V magnitude and (5) its error, (6) the weighted I magnitude and (7) its error, and (8) the number of frames on which the star was detected. The table contains data for 20,346 stars.

Figure 1 presents a V-band image of the cluster. The XY coordinate system is the same as presented in Table 2, and several stars from this table are identified on the image. We used the IRAF task ELLIPSE to find the center of M54 by fitting ellipses to a heavily boxcar-smoothed, short exposure image of M54. The mean center in the coordinate system shown in Figure 1 is $(X, Y) = (726, 806)$ pix. This agrees with a visual estimate of the center to within 2 pixels.

Figure 2 shows differences between our final bright-sequence photometry set and the SL95 photometry of their Table 1. Figure 1a indicates the presence of a small V magnitude difference as a function of magnitude. Since it is not clear whether the fault lies with the SL95 or the present photometry, and since the effect is small and not well defined, we have not corrected for it in our 4.0-m photometry. Note that the stars with $V < 17$ mag are not used directly in any of the results derived in this paper – both the RR Lyrae variables and the stars used to calibrate the deep M54 photometry are fainter than this level. On the whole, the transformation to the photometric system of SL95 was successful.

3.3. Deep Photometry of M54

We followed a similar procedure to obtain photometry from the deep frames of M54. We trimmed the deep frames to cover the same region of the sky as the shallow frames, thus no spatial extrapolation of the photometric transformation (§3.1) is required in this section. We then computed instrumental stellar magnitudes using the DoPHOT program, and matched the instrumental photometry of spatially coincident stellar images within each pointing (contemporaneous set of V, I, I images).

An (X, Y) coordinate transformation brought the coordinate frame of each instrumental photometry list onto the same coordinate system used for the bright M54 sequences (see Figure 1). We then used the equations shown in §3.1 to transform these instrumental magnitudes onto the standard VI magnitude system defined by the data in Table 2. The RMS scatter in the transformations were typically $\sim$0.020 mag, and the transformations always employed $>$900 stars, and spanned the entire region of color–X–Y space.

We then combined the photometry of spatially coincident stars in the resulting photometry lists. Experiments showed that the assembled photometry of the five pointings with the best seeing (median = 1.1 arcsec) produced the CMD with the tightest sequences; including the poorer seeing data degraded the photometric quality in the crowded inner regions of the cluster. Therefore, we used data only from the five pointings with the best seeing. For each star appearing in three or more of the five photometry lists, we computed the weighted mean V and I magnitudes (weighting the individual magnitudes by their DoPHOT errors). We also computed the standard error of the mean of the 3–5 V or I observations, which we treat as the error in the V or I magnitude. These final photometric data are presented in Table 3, in the same format as Table 2. There are 30,081 stars in this data set.

3.4. Deep Photometry of Sgr

We did not obtain short exposure frames of the Sgr field, so we transformed the MSTO instrumental photometry directly onto the standard VI photometry presented in Table 2 of SL95. We began by trimming the 4.0-m images down to a 8.2 arcmin FOV centered on the SL95 Sgr field. This left a relatively small (0.7 arcmin) strip around the outside of the image over which the photometric transformation's spatial fit would be extrapolated. The trimmed field lay near the physical corner of the CCD, and in some frames the image quality in the South-West corner was sufficiently poor that we masked out these regions. Thus, the final photometry data set is deficient in stars in the that corner of the field.

We then computed instrumental magnitudes using the DoPHOT program, and matched the instrumental photometry of spatially coincident stellar images within each pointing. We performed the (X, Y) coordinate transformation to bring the coordinate frame of each instrumental photometry list onto the XY coordinates of the image shown in Figure 3 (we also transformed the XY coordinates of the short exposure data from SL95, their Table 2, onto this coordinate system; we present those data in Table 4 for convenience). We then used the equations shown in §3.1 to transform these instrumental magnitudes onto the standard VI magnitude system defined by the short exposure data in Table 2 of SL95 (our Table 4). The RMS scatter in the transformations were typically $\sim$0.024 mag, and the transformations always employed $>$220 stars.

We then combined the photometry of spatially coincident stars in the resulting photometry lists. In this case, experimentation showed that including data from the six pointings with the best seeing (median = 1.2 arcsec) produced the best CMD. We used only these six pointings in our final analysis. For each star which appeared in four or more of the six photometry lists, we computed the weighted mean V and I magnitudes and their errors as in §3.2. These data are presented in Table 5, in the same format as Table 2. There are 10,147 stars in this data set.

Figure 4 shows differences between our final Sgr photometry and the SL95 photometry listed in Table 4. The rise in ΔV at $V < 18.5$ seen in panel (a) mimics that seen in the bright sequence data (Figure 2(a)). However, only stars with $I > 17.3$ ($V \gtrsim 18.3$) were used in the photometric transformation, so the affects of this rise are negligible. Overall, the transformation to the SL95 VI system was successful.

3.5. Photometry in the Bulge Control Field

No previous photometry exists for our bulge control field, so we can not transform our instrumental photometry to standard VI photometry using the method employed in the previous subsections. Fortunately, the sky conditions on 1995 June 29 were photometric, and we obtained five short exposure VI image pairs of M54 which spanned the airmass and Universal Time of the control field observations. We used these observations to derive photometric transformation equations which we applied to the deep observations of the control field – essentially, we used the M54 stars as secondary photometric standards. In practice, the airmass-dependence of the transformation was minimal, and only a color transformation was employed:

$$m_f - M_f = \alpha_f + \beta_f \ (V - I),$$

where m and M are the instrumental and standard magnitudes, respectively, and the subscript f refers to the filter, V or I. The coefficients α_f and β_f were derived from the short-exposure M54 observations, and applied (below) to the control field observations. Application of the method to a single, short-exposure V image of M54 taken during the same time interval reproduced the standard magnitudes accurately, thus confirming the validity of this transformation method and the photometric quality of the night.

We trimmed the control field images down to a 10.7 arcmin square FOV to avoid poor image quality near the corners of the frames. We then computed instrumental magnitudes using DoPHOT, and matched the instrumental photometry of spatially coincident stellar images within each pointing. We performed the $(X,\ Y)$ coordinate transformation to bring the coordinate frame of each instrumental photometry list onto the coordinate system of the frame with the best seeing. We then applied the photometric transformation equations described in the previous paragraph to convert the instrumental magnitudes to the standard VI system. Next, we combined the photometry of spatially coincident stars in the four photometry lists (one list per pointing), retaining only the stars which appeared in three or more of the lists. We computed the weighted mean V and I magnitudes and their errors as in §3.2. These data are presented in Table 6, in the same format as Table 2. There are 7917 stars in this data set. The median seeing of the frames employed was 1.2 arcsec.

The purpose of the control field is to quantify the positions of foreground bulge stars in the M54 (on-cluster) and Sgr CMDs. The photometry provided herein should be adequate for that purpose, despite the fact that a different technique was used in the photometric transformations, relative to the M54 and Sgr fields. We emphasize that no such difference exists between the M54 and Sgr photometry. Furthermore, we can use the MUSKKK bulge control field (see Table 1) as an additional control sample.

4. Color Magnitude Diagrams

The color-magnitude diagrams (CMDs) of our M54 and Sgr fields can be used to determine the ages of these systems by using techniques such as isochrone fitting and comparison with CMDs of clusters of known age. However, the foreground bulge population complicates the interpretation of the Sgr CMD, while the bulge and Sgr populations together complicate the CMD of M54. In this section, we will attempt to simplify the CMDs by statistically subtracting away the contaminating populations, and where appropriate, by excluding stars at certain radii.

4.1. The Sagittarius CMD

We begin with the simpler case of Sgr. To get the widest possible range in magnitudes, we have combined the deep photometry from Table 5 with the shallower photometry from SL95 (our Table 4); the splice occurs at $V = 18.7$. Because the SL95 data covers a slightly smaller field of view, the number of bright stars is slightly underrepresented relative to the number of faint stars. Figure 5(a) shows the CMD of the 9,540 stars which have errors in V and $V - I$ less than 0.10 and 0.14 mag, respectively.

Figure 6(a) shows this data again, zoomed in on the MSTO region. Panel (b) of this figure presents the CMD of our control field data (from Table 6) which has errors in V and $V - I$ less than 0.10 and 0.14 mag, respectively. Since we have no photometry of the brighter stars in this field, the CMD is cut off above $V = 18$ mag. The bulge MSTO is seen at $V \approx 19$ mag and $V - I = 0.75$, while the ridgeline of stars extending to fainter magnitudes and redder colors marks the bulge main sequence. This same pattern is evident in panel (a), albeit with fewer stars, since the Sgr data was derived from a smaller field of view (8.2×8.2 versus 10.7×10.7 arcmin). The bulge MSTO region appears to have a narrower color width in panel (a) than in panel (b), suggesting that differential reddening across the control field may be larger. Also, the bulge MSTO region appears to be about 0.1 mag bluer in panel (b), suggesting a lower mean reddening in that field. Fortunately, we can use the MUSKKK control field photometry as a "second opinion" on the foreground contamination. The

MUSKKK data extends to brighter magnitudes, but has a lower signal-to-noise ratio at a given faint magnitude.

The excess of Sgr stars in Figure 6(a) over the bulge contribution is obvious. The primary Sgr MSTO occurs near $V \approx 21.4$ and $V - I = 0.75$, well separated from most of the bulge stars. A large fraction of the Sgr stars seem to form a population which turns redward onto the subgiant branch (SGB) at $V = 21.1$ mag, though a significant number of stars contribute to one or more MSTOs at brighter magnitudes. The SGBs of the Sgr populations can be seen as slight density enhancements crossing the bulge main sequence, and turning up onto a sparsely-populated RGB. The Sgr red clump is at $V = 18.3$ and $V - I = 1.13$ mag. Also notable in panel (a) are the stars brighter and bluer than the Sgr MSTO. These stars are not present in the comparison fields, and may represent either a very young population or blue straggler stars in Sgr (MUSKKK, Bellazzini et al. 1999a).

We can statistically subtract the stars in panel (b) from those in panel (a) to get a clearer picture of the various stellar populations in Sgr. First, we need to characterize two parameters, the reddening difference between the fields, and the density normalization. SL95 estimated $E(V - I) = 0.18 \pm 0.02$ in our Sgr field, using the shape and color of the Sgr RGB. We have no corresponding reddening estimate from our control field. The reddening maps of Schlegel et al. (1998) give values of $E(V - I) = 0.19$, 0.10, and 0.13 at the positions of our Sgr field, our control field, and the MUSKKK control field, respectively. The $E(B - V)$ values from Schlegel et al. have been converted to $E(V - I)$ using the relations of Dean et al. (1978).

We check these reddening differences by comparing the color histograms of stars near the MSTO in the three fields. We extract from the CMD of each field all stars with $16.0 < V_{VI} < 18.0$ mag, where V_{VI} is the reddening-independent parameter defined as $V - 2.6\ (V - I)$ by Stanek et al. (1994). The red wing of the color histogram for the Sgr field is contaminated by Sgr RGB stars. We therefore compare the colors of the blue edge of the histograms. They coincide when our control field data are reddened by 0.07 mag, and when the MUSKKK control field data are reddened by 0.05 mag. These correspond well with the reddening differences from the Schlegel et al. (1998) dust maps. We therefore adopt $\Delta\ E(V - I) = 0.08$ for our control field, and 0.04 mag for the MUSKKK control field.

The density of bulge stars in each field can be estimated from the number of stars in a region along the bulge main sequence, around $V = 21.8$ and $V - I = 1.3$ mag. The relative numbers are used to normalize the control field samples to the Sgr field. Since the signal-to-noise and detection-completeness functions vary between the different fields, and since the tightness of the bulge sequence may be affected by reddening variations within a field, these normalizations are only approximate, and we treat them as adjustable parameters in our subtractions.

Figure 6(c) shows the data from panel (a) after the subtraction of a subset of stars from panel (b) which had been reddened by 0.08 mag. The subtraction procedure involved identifying for a given star in the control CMD the star in the Sgr CMD with the smallest "distance", ρ, and removing this star from the Sgr CMD. We define ρ as

$$\rho = (\ (V_c - V_s) + \alpha\ \{\ (V - I)_c - (V - I)_s\}\ )^{0.5}.$$

The subscripts c and s refer to the control and Sgr data sets, respectively; V and $(V - I)$ are the corresponding magnitude and color; α accounts for the aspect ratio of the CMD. We found that $\alpha = 3$ produced good results.

Figure 6(c) shows the Sgr populations much more clearly. In particular, the SGB region is more clearly defined. It shows a large vertical spread which may indicate multiple epochs of field star formation in Sgr, or a long duration of continuous star formation (see §8). The population of bright, blue stars is also clearly shown.

We also performed subtractions employing the MUSKKK control field data. Subtractions using the preferred reddening shift of 0.04 produced satisfactory results, but tended to oversubtract stars on the Sgr RGB, and leave bulge stars $V < 20$ and $V - I < 0.8$ mag. A reddening shift of 0.03 mag produced a much better subtraction, which we show in Figure 6(d). The same features appear as in panel (c), confirming that they are real features of the Sgr populations. A slight excess of unsubtracted bulge stars remains below and to the right of the Sgr SGB. The knot of stars at $V = 18.7$ and $V - I = 0.85$ in both panels (c) and (d) is spurious: the result of the signal-to-noise discontinuity at the joint of the deep and shallow Sgr photometry sets.

4.2. The M54 CMD

As in the previous section, we merged the deep and shallow M54 photometry sets from Tables 3 and 2, respectively. The splice was made at $V = 19.0$. The field sizes are the same, so the relative numbers of bright and faint stars should be consistent across the boundary. Figure 5(b) shows the CMD of the 28,950 stars which have errors in V and $V - I$ less than 0.10 and 0.14 mag, respectively. The features of the M54 CMD, including a blue HB, a more vertical RGB, and the MSTO, are superimposed on the Sgr and bulge features seen in panel (a). Figure 7(a) shows the M54 data again, zoomed in on the MSTO region. Panel (b) of this repeats the Sgr data from 6(a).

It is useful to plot the CMD of stars with positions inside an annulus around the globular cluster center, where the inner radius is chosen to minimize photometric errors due to crowding, and the outer radius is chosen to minimize contamination by field stars. Figure 7(c) shows the CMD of M54 stars lying within 300–600 pix (2.2–4.3 arcmin) of the cluster center, $(X, Y) = (726, 806)$ pix. Note that this region is mostly contained within the area covered by the SL95 photometry, so the photometric transformations discussed in Secs. 3.2 and 3.3 are particularly trustworthy. The sequences associated with M54 become clearer in this panel, though there is still a strong contribution from the Sgr and bulge populations.

In Figure 7(d) we present the result of statistically subtracting the data in panel (b) from that in panel (c). We made no reddening correction, since SL95 found identical reddenings from the shape and position of the RGBs, and since the centroids of the red clumps coincide to within 0.01 mag. The areas of these two fields are nearly identical, but normalization factor computed from counts of main sequence bulge stars called for a reduction in the Sgr contribution by about 10%. This is explained by the enhanced crowding in the M54 field: some fraction of the faint bulge stars will be overwhelmed by bright stars belonging to the cluster, and thus not detected. However, to eliminate successfully the Sgr population in panel (d) required an enhancement of the Sgr field contribution by 20%, and even then, traces of the brightest Sgr MSTO, SGB, and RGB remain. This supports the idea raised in SL95 that the young, metal-rich component is concentrated toward the cluster. We will pursue this question in §10.

5. Variable Stars

The large number of CCD frames which we have accumulated in our study of M54 gives us an excellent opportunity to study the variable stars in this cluster. The most recent survey for variable stars in this cluster was the photographic work of Rosino & Nobili (1959, hereafter referred to as "RN59"). They identified 82 variable stars in this cluster (see §5.4).

In our investigation, we assume no *a priori* knowledge of the variable stars found by RN59 – we begin from scratch. Table 7 describes the five observing runs involved in our study. Columns 1–5 give the designation and the UT dates of the run, the telescope and CCD employed, and the field of view in arcmin. Columns 6 and 7 list the number of frames obtained during that run in the V and I bands, respectively. Different plot symbols will be used to distinguish data taken during the different runs; these symbols are defined in the last column on Table 7.

Instrumental photometry for each of the individual frames was computed using DoPHOT, and was then transformed onto the (X, Y) coordinate system defined by the stars on the short-exposure CTIO 4-m images (see §3.2, Figure 1). Then it was transformed photometrically onto the bright sequence photometry of Table 2 as described in §3.2. Photometry from the 15 VI frame pairs with the best seeing were assembled, i.e., we spatially matched stars from the different frames and combined the photometry for each star using an error-weighted mean.

For each star, we also computed the Welch & Stetson (1993) variability index, $I_{\rm WS}$, to characterize the likelihood that the star is variable. The value of $I_{\rm WS}$ is computed for each star from the individual V and I magnitudes and their errors. For a non-variable star, $I_{\rm WS}$ approaches zero as the number of observations becomes large, whereas for a variable, $I_{\rm WS}$ approaches a constant value.

The morphology of the points in the $(I_{\rm WS}, V)$ plane led us to classify stars as probable and possible variables. When plotted on the cluster CMD, it became clear that three classes of variables are present. Most of the variable candidates fell near the middle of the cluster horizontal branch, and are very probably RR Lyrae variables in M54 or in Sgr. A number of candidates lay near the tips of the Sgr and M54 red giant branches. A few other variables scattered around the CMD, mostly blue-ward of $V - I = 1$, suggesting that they are RR Lyrae or eclipsing binaries in the foreground field.

Our main goal in this study is to use the M54 RR Lyrae stars to define the distance and reddening to the cluster. We therefore focused closely on the stars near the M54 HB, in the region $V - I < 1.0$ mag, $17.8 < V < 18.9$ mag. We found 82 such stars with $I_{\rm WS} > 30$. We also considered possible foreground stars in the region $V - I < 1.0$ and $V < 17.8$ mag. Eleven such stars were found with $I_{\rm WS} > 50$. Plots of magnitude as a function of time for these 93 stars showed that, almost universally, they had periods shorter than about a day, consistent with their being RR Lyrae or contact binary stars. We also searched for variability in the bright, red stars with $V - I > 1.4$, $I < 15$, and $I_{\rm WS} > 10$. We found 26 candidates, whose characteristics will be discussed in detail in §5.2.

5.1. Short Period Variables: the RR Lyrae

In an effort to determine the types of variable stars present among the 93 blue, short-period variable star candidates discussed above, we attempted to find the period of each star and create the star's mean light curve. Classical period finding techniques such as Fourier transforms or Phase Dispersion Minimization perform poorly, if at all, when the data are sparse and irregularly spaced. We therefore used the method of template fitting described in Layden (1998) and Layden et al. (1999).

Briefly, a set of template light curves representing the types of variable stars expected are fit to the data, which are folded by a set of periods ranging over the expected values. At each period in the series, the observed V-band data are folded and fitted by the each of the templates (three free parameters are used: magnitude zero-point, phase zero-point, and magnitude amplitude), and the χ^2 of the fit is computed. Small values of χ^2 indicate possible periods. For each template, a plot of χ^2 as a function of period allows us to select the most probable periods.

In practice, we used the six templates shown in Layden (1998) to represent ab-type RR Lyrae (RRab, fundamental mode pulsators). We also employed templates for a c-type RR Lyrae (RRc, first-overtone pulsator)[4], a cosine curve, a curve derived from the Fourier coefficients of Rucinski (1993, his Figure 1) to represent a W Ursae Majoris (W UMa) contact binary, and a schematic template of a detached (Algol) eclipsing binary. While the binary star templates do not represent the entire range of light curves displayed by these variables, we think they are sufficient to identify eclipsing binaries. We will not derive useful information from these stars, so we are content to cull them out of the RR Lyrae sample. Indeed, given the limited number of observations, it is difficult to derive reliable periods for Algol-type binaries, and it is likely that some exist in our field which have not been detected.

We fit the templates over periods ranging from 0.2 to 1.0 days, increasing the period by 0.03% from one step to the next. If no suitable minima were found, we searched on the period interval 1–3 days, and in some cases, 0.1–0.2 days. Periods longer than 3 days are evident in the time-magnitude plots. For each variable, we inspected the light curves produced with periods from the best ten or so minima. In almost all cases, the period with the smallest χ^2 value in the array of periods and templates produced the light curve most pleasing to the eye, and this period and template were adopted. In most cases, changing the period by 0.00005 days resulted in a noticeable degradation of the light curve, but in many cases a nearby minimum produced a satisfactory light curve as well.

Periods could not be determined for 32 stars. Possible reasons include, (1) photometric errors due to crowding produced light curves with unacceptable scatter, (2) the star has multi-periodic behavior, (3) the star undergoes changes in light curve shape or amplitude (e.g., the Blazhko effect) from cycle-to-cycle, and (4) the templates we employed did not reflect the star's actual light curve shape. Upon inspection of the images, 28 of these stars were seen to have stars very close by; the stellar images blended on poor-seeing frames, and separated on good-seeing frames, mimicking variability. We do not consider these stars farther. The three candidates which *do* appear to be true variables, but for which we could not determine a period, are listed in Table 8 as SV1–3.

We were able to derive periods for 60 of the 93 short-period variable candidates. We obtained photometry for 11 more known variables after identifying them with the finder chart from RN59. Figure 8 shows the V-band light curves of these stars, folded by their adopted periods, and with the preferred template overplotted. In Table 9, we present the data derived from these stars: the first column gives the variable star name; $X_{\rm pix}$ and $Y_{\rm pix}$ are the positions on

[4] The RRc template is an average of ten high-quality light curves of RRc stars in M68 (Walker 1994).

Figure 1; Per is the period in days; Typ is the variable type; $\langle V \rangle$ and $\langle I \rangle$ are the intensity-mean magnitudes in V and I; $V_{\rm max}$ and $V_{\rm min}$ are the V-band magnitudes at maximum and minimum light; $E_{\rm max}$ is the epoch of maximum light (Heliocentric Julian Date - 2,449,000 days); $N_{\rm obs}$ is the number of V frames on which the variable was measured; $N_{\rm tmp}$ is the template number used in the adopted fit; rms is the root-mean-squared scatter of the data points about the fit; $(V-I)_{\rm min}$ is the magnitude-mean $V-I$ color at minimum light, defined as $0.5 <$ Phase < 0.8 (Sturch 1966); $\epsilon(V-I)_{\rm min}$ and $N_{\rm min}$ are the standard error of the mean in $(V-I)_{\rm min}$, and the number of observations used in its computation. The a mark in the final column of Table 9 indicates that a note regarding that star is available in the Appendix. The quantities $\langle V \rangle$, $V_{\rm max}$, $V_{\rm min}$, $E_{\rm max}$, and rms were derived from the fitted light curve, the others are from the observed data points.

Table 9 contains 71 stars, 60 of which are RRab, and 7 are RRc. V01 appears to be a Population II Cepheid, and V02 is probably an Anomalous Cepheid (see Appendix). There is also one probable W UMa contact binary (EW, V99), and one probable Algol detached binary (EA, V91).

5.2. Long Period Variables

In addition to the RR Lyrae and other short period variables, we detected 26 bright, red variable star candidates in our data. Figure 9 shows a plot of magnitude versus time for a typical example, V25. The magnitudes vary slowly, if at all, during the course of a single run, but jump noticeably between runs, indicating that the stars have long periods. Given their colors and periods, they are very likely to be Irregular, Semi-Regular, or Mira type pulsating variables. Indeed, three of our Long Period Variable (LPV) candidates appear in the Sawyer-Hogg (1973) catalog (V25, V66, and V71), and all three are classified as "SR" (Semi-Regular) therein. The fourth SR listed in Sawyer-Hogg (1973), V64, is outside our field of view. Careful inspection of all the magnitude-time plots showed that eight of the candidates were probably non-variable stars whose photometry had been compromised by crowding in the densely-packed cluster center.

The 18 remaining LPV candidates are listed in Table 10, along with their (X,Y) positions in Figure 1, their arithmetic mean V magnitudes and $V-I$ colors, the maximum difference in V magnitudes observed $(\Delta V_{\rm min}^{\rm max})$, and in some cases a comment. Stars with larger values of $\Delta V_{\rm min}^{\rm max}$ are likely to be true LPVs, and the probability decreases with decreasing amplitude, as noted in the comments column. Additional uncertainty is encountered in some stars in crowded environments, where increased photometric errors are incurred.

We emphasize that our list of LPV stars is likely to be incomplete for several reasons. First, there may be some bulge LPV stars in the field which are brighter than our saturation limit. Second, the distribution of our observations in time is not ideal for detecting variables with periods longer than a few days; it is possible that some true variables did not vary enough over the course of our observations to be detected. We also note that the mean photometry listed in Table 10 is only approximately representative on the true values, since the stars' light cycles are poorly sampled. Clearly, long-term observations of the LPVs would be useful.

5.3. Faint Variables from Deep Exposures

For completeness, we also searched the deep M54 and Sgr frames for variable stars. The poor time sampling, 8–9 frames over two nights, means that we will not be able to determine periods, and that significant numbers of actual variables may go undetected. Nevertheless, it is interesting to identify any variables as motivation for more complete study in the future.

In the M54 frames, we detected two eclipsing variables which we designate V116 and V117. Table 11 gives their (X,Y) positions in Figure 1, their arithmetic mean V magnitude and $V-I$ color, their maximum difference in V magnitude $(\Delta V_{\rm min}^{\rm max})$, and a comment. Time series data for these stars is found in Table 12.

In the Sgr frames, we detected two more variables, which we designate Sgr1 and Sgr2 (see Table 11). Sgr1 has the right light curve shape, magnitude, and color to be an RR Lyrae star in the Sgr field. Sgr2 is an eclipsing binary. Time series data for these stars is found in Table 12.

5.4. Previously Known Variable Stars

The most recent, significant study of the variable stars in M54 is the photographic work of RN59. Results from this and other works are summarized in Sawyer-Hogg (1973, hereafter "SH73"), and have been updated by Christine Clement[5]. SH73 lists 82 variable stars in M54, but less than half of these stars have well determined periods or variability types. Finder charts of variables V1–V82 are presented in RN59.

Of these 82 stars, 13 are outside our field of view (V8, V9, V10, V11, V13, V16, V17, V18, V19, V24, V64, V68, and V70). We could not identify two stars (V21 and V26; see Appendix). Five stars appeared not to be variable, despite being identified positively on the RN59 charts (V20, V22, V27, V53, V72; see Appendix).

We did detect variability in the remaining 62 stars, three of which were long period variables. We could not determine periods for the six stars (see §5.1; V67, V69, V73, V75, V79, V81; see Appendix). However, we did obtain good periods and light curves for the remaining 53 known cluster variables.

We also detected 35 new variables stars. Twenty of them have short periods, and appear in Table 9, while fifteen have long periods, and appear in Table 10. We also detected five new stars which appear to be variable, but for which we could not obtain a period or light curve (V116, V117, and SV1–3 in Table 8). We begin naming the new variables at V83. This brings the total number of known or suspected variables toward M54 to 120.

We present the magnitude data as a function of time in Table 12. The columns contain: (1) the star name, (2) the reduced Heliocentric Julian Date (HJD - 2,449,000) in days, (3) the adopted phase, (4) the V magnitude and (5) its error, (6) the I magnitude and (7) its error, and (8) the symbol with which the data are plotted in Figures 8 and 9.[6] Epochs for which I frames were not obtained are assigned the values 88.888 in columns (6) and (7). In Figure 10, we provide a "postage stamp" finder chart for each variable in our field of view. Figure 11 shows the mean magnitudes and colors of the variable stars near the HB superimposed on the cluster CMD.

[5]The updated electronic catalog is available upon request from Dr. Clement (cclement@astro.utoronto.ca).

[6]This code is a Super-Mongo point type. The first digit indicates the symbol shape and run number (see Table 7): 3 = *triangle* = Run1, 4 = *square* = Run2, 5 = *pentagon* = Run3, 9 = *circle* = Run4. The second digit indicates whether the seeing was better or worse than the median seeing (3 or 1, respectively). The exception to this rule is code=41, which indicates Run5.

6. Cluster Properties Derived from the RR Lyrae Stars

We can derive two important quantities from the RR Lyrae stars in M54: the cluster distance modulus, and the foreground reddening. The former comes from the observed apparent magnitudes of cluster RRab and RRc stars, while the latter is derived from all the RRab stars available, be they members of the cluster, of Sgr, or of the Galactic foreground. Before proceeding, we must attempt to determine which stars are members of which population. We will discuss a number of other interesting results regarding the HB of M54 as we progress toward measuring its distance and reddening.

6.1. RR Lyrae Membership in Sgr and M54

Assigning membership in this three-population case is more complicated and prone to uncertainty than the usual two-population case (cluster versus field). The foreground RRL stars will be uniformly scattered over the field of view, and display a range of apparent magnitudes. The Sgr member RRL will also be scattered over the field, but will all have very similar magnitudes. The M54 RRL members will be radially concentrated toward the cluster center, but will have the same magnitudes as the Sgr RRL, since the distances to M54 and Sgr are indistinguishable (SL95).

We begin by trying to identify RR Lyrae stars in the foreground field. In a plot of V magnitude as a function of radius, there are only four clear outliers from the swath of stars defining the M54/Sgr locus: V12, V55, V65, and V93. V55 is 0.4 mag fainter than the cluster RR Lyrae, while the other three are significantly brighter. The four RR Lyrae about 0.2 mag brighter than the clump of M54/Sgr RRab in Figure 11 all lie within 1.3 arcmin of the cluster center, so are almost certainly cluster members. Their enhanced brightness is probably due to image blending in this croweded region.

Having culled the field RRL from our sample, we now attempt to separate M54 members from Sgr members. One way to accomplish this is by comparing the radial distributions of the various HB components in our CMD. Figure 12 shows the cumulative radial distributions of the blue HB stars (BHB), RR Lyrae stars (RRL), red clump stars (RC), and stars belonging to the "anonymous red HB" (AHB) discussed by SL95. Here, we defined AHB stars as those with $18.17 < V < 18.33$ and $0.75 < V-I < 0.90$

mag. SL95 showed that the BHB stars belong to the cluster, and that the RC stars belong to Sgr, while the source of the AHB stars was tentatively attributed to Sgr. In Figure 12, it is clear that the AHB star distribution follows that of the RC stars, again suggesting that the AHB stars belong to Sgr. Meanwhile, the RRL are distributed more like the BHB stars, indicating that most of the RRL belong to M54.

The RRL distribution does flatten a bit at $R > 500$ pix, suggesting an additional contribution of stars at large radius relative to the BHB stars. We experimented with subtracting random samples of RRL stars from the RRL distribution outside this radius, and found that the RRL and BHB distributions matched better when 3–9 RRL were removed. Given that RRL are known to exist in the Sgr field (e.g., MUSKKK, Alard 1996), it is reasonable to believe that these excess stars are members of Sgr. The region outside $R = 500$ pix comprises about $^2/_3$ of the area of our frame, so the number of Sgr RRL in our entire frame is probably between 5 and 14. For comparison, MUSKKK found 6–7 Sgr RRL in a 15×15 arcmin field about 2.4 degrees away, in a lower density region of Sgr according to the Ibata et al. (1994) isopleths.

6.2. The Period–Amplitude Diagram and Oosterhoff Type

Is it possible to determine exactly which RRL belong to Sgr and which belong to M54? One means may be through the stars' positions in the Period-Amplitude diagram. It has long been thought that stars with lower metal abundances shift to longer periods in this diagram (Sandage 1993 and references therein). Since M54, at $[Fe/H] = -1.55$ to -1.79, is significantly more metal-poor than the old Sgr population responsible for the RRL ($[Fe/H] \approx 1.3$), we expect the M54 RRL to have systematically longer periods than the Sgr RRL. Sandage's (1982, Eqn. 2) relation predicts the shift to be $\log P \approx -0.06$ dex.

Figure 13 shows the positions of our RRL (the four field RRL are plotted as squares) in the Period-Amplitude diagram. The majority of stars seem to follow the relation derived by Clement & Shelton (1999) for M3, though there might be a slight difference in slope or zero point. Ten of the RRab seem to follow the relation expected for Sgr from the Sandage period-shift relation (dotted line), and two of these belong to the field (V55 and V93). Eight Sgr RRL agrees well with the estimate of 5–14 from the flattening of the RR Lyrae cumulative radial distribution. However, two points conflict with this interpretation. First, only three of the eight stars lie more than 500 pix from the cluster center, whereas we would expect the number to be 5–6 if the population is distributed uniformly across the field. The radial distribution of the eight stars is like that of the M54 stars. Second, if the period shift is due solely to metallicity alone, presence of six or so stars shifted to *longer* periods would suggest the presence of an additional *metal-poor* field population in Sgr, at $[Fe/H] \approx -2.1$ dex. While Bellazzini et al. (1999a, 1999b) suggest that a minor fraction of the Sgr field has this abundance, (*i*) it is not clear whether those stars produce RRL, and (*ii*) most of the longer-period RRL in Figure 13 lie within 500 pix of the cluster center, and thus probably belong to M54.

Clement & Shelton (1999) provide a different interpretation of the period-shift effect. They claim that the shift is not a continuous function of metallicity, but is instead a two-valued distribution directly related to the Oosterhoff dichotomy. That is, all Oosterhoff type-I clusters follow the relation defined by M3, and all Oosterhoff type-II clusters follow the relation defined by M9. They obtained narrow sequences of stars in the Period-Amplitude diagram by identifying and rejecting all RRab with "peculiar" light curves (e.g., the Blazhko effect) using the compatibility test of Jurcsik & Kovacs (1996). The peculiar stars generally lay below the relation defined by the "normal" stars.

Clearly, most of the RRab in Figure 13 lie near the M3 relation, suggesting M54 is a Oosterhoff type-I cluster. The mean periods of the RR Lyrae confirm this: $\langle P_{\rm ab} \rangle = 0.579 \pm 0.007$ and $\langle P_{\rm c} \rangle = 0.342 \pm 0.016$ days. For comparison, the average values for Oosterhoff type-I clusters (OoI) are $\langle P_{\rm ab} \rangle = 0.55$ and $\langle P_{\rm c} \rangle = 0.32$ days, while for Oosterhoff type-II clusters (OoII) they are $\langle P_{\rm ab} \rangle = 0.64$ and $\langle P_{\rm c} \rangle = 0.37$ days (Smith 1995, Table 3.2).

According to the picture of Clement & Shelton (1999), the RR Lyrae stars in Sgr and M54 should both follow the M3 relation (presumably being more metal-rich than M54, Sgr is also a OoI system), and we would expect most of the stars lying near the dotted line in Figure 13 to have peculiar light curves. Unfortunately for us, the Jurcsik & Kovacs (1996) compatibility test involves high-order Fourier decompositions of the RRab light curves, and we do not have enough data points per star to provide reliable determinations of the stars' Fourier coefficients. Still, one might expect stars with light curves which repeat faithfully from cycle to cycle to produce a relatively small rms scatter about a best-fit template, while stars with variable light curve shapes or amplitudes would tend to have larger values of rms. In the Clement & Shelton view, we would then expect the low-rms stars (solid circles) to lie on the M3 relation, while the high-rms stars (open circles) would lie below it. This does not appear to be the case in Figure 13.

However, the six longer period RRab in Figure 13 *do* have a reasonable explanation in the Clement & Shelton (1999) picture. Those authors note three stars in M3 with normal light curves which obey the OoII (M9) relation. They suggest that these stars evolved off the blue HB and through the instability strip at higher luminosity, and thus longer period, than the other M3 variables, which lie closer to the zero-age HB. Since the blue HB of M54 is more heavily-populated than in M3, it is reasonable to expect a higher fraction of OoII-like variables in M54.

At this stage, the uncertainties in our data do not allow us to support firmly either the Sandage (1982) or Clement & Shelton (1999) explanation of the period-shift effect. Nor we can use the Period-Amplitude diagram to identify RRL which are members of the Sgr field population. An observational dataset with more observational epochs would allow us to do reliable Fourier decompositions, and clarify the Period-Amplitude diagram using the Jurcsik & Kovacs (1996) compatibility test. Below, we show that the inclusion of eight or so Sgr stars in the M54 sample does not change significantly the derived value of $\langle V(\mathrm{RR})\rangle$ for M54.

6.3. $\langle V(\mathrm{RR})\rangle$ of M54

After removing the four field RRL from our sample, we determined the mean V magnitude of the 63 remaining RRL using a $3-\sigma$ clipping algorithm. We obtained $\langle V(\mathrm{RR})\rangle = 18.17 \pm 0.01$ mag, with an rms of 0.06 mag. We obtain the same value if we remove the eight candidate Sgr RRL which lie near the dotted line in Figure 13. The systematic uncertainty in the photometric zero point is about 0.02 mag (SL95). In SL95, we estimated $\langle V(\mathrm{RR})\rangle = 18.17 \pm 0.05$ mag from 16 RRL which were poorly sampled in light curve phase.

Most researchers studying the change in RR Lyrae absolute magnitude, $M_V(\mathrm{RR})$, with $[Fe/H]$ find a slope of $\sim$0.23 mag dex^{-1} (Chaboyer 1999). If we assume that M54 and Sgr are equidistant, and that they have a metallicity difference of $\sim$0.5 dex, any Sgr variables should be about 0.12 mag fainter than their counterparts in M54. If 15% of our RRL sample are members of Sgr, the $\langle V(\mathrm{RR})\rangle$ estimate for M54 quoted above may be systematically too faint by $\sim$0.02 mag.

6.4. Reddening toward M54

In SL95, we determined the reddening toward M54 to be $E(B-V) = 0.13 \pm 0.02$ mag from the shape and color of the M54 red giant branch (Sarajedini 1994). The dust-based extinction map of Schlegel et al. (1998) gives $E(B-V) = 0.15 \pm 0.02$ mag, while the map of Burstein & Heiles (1982), which is based on H I gas and galaxy counts, gives 0.14 ± 0.02 mag.

We can obtain another, independent reddening estimate from the color of the RRL. MUSKKK showed that the intrinsic colors of ab-type RRL, measured at minimum light (phases between 0.5 and 0.8), are universal: $\langle (V-I)_{0,min}\rangle = 0.58 \pm 0.03$ mag, independent of metallicity and period. From Table 9, we find the apparent color of the 29 RRab in M54 with four or more observations during minimum light to be $\langle (V-I)_{min}\rangle = 0.76 \pm 0.01$ mag. The value does not change if we include stars with fewer data points at minimum light. The color distribution is very narrow, the standard deviation being 0.03 mag, and there is no correlation with obvious parameters such as period or distance from the cluster center.

The color excess toward M54 is thus $E(V-I) = 0.18 \pm 0.04$ mag, where we have included a systematic error of 0.02 mag in the mean apparent color due to uncertainty in the photometric zero point (SL95). Using the relations of Dean et al. (1978), this reddening corresponds to $E(B-V) = 0.14 \pm 0.03$ mag. Thus the RRL-based result is in excellent agreement with the three previous reddening estimates. In the remainder of this paper, we adopt $E(V-I) = 0.18 \pm 0.02$ mag and $A_V/E(V-I) = 2.6$.

6.5. Distance Modulus

In order to compute the distance modulus to M54, we must assume a value for the RR Lyrae absolute magnitude, $M_V(\mathrm{RR})$. Chaboyer (1999) reviews the recent, post-*Hipparcos* results. We adopt his relation

$$M_V(\mathrm{RR}) = 0.23\ ([Fe/H] + 1.6) + 0.56.$$

The slope is fairly reliable, but the zero-point has an uncertainty of 0.12 mag stemming from lingering disagreements between the various methods used to obtain M_V(RR). We note that a systematic change in the zero-point affects all our results in concert, e.g., affecting the relative ages less than the absolute ages.

Adopting $[Fe/H] = -1.79$ for M54 (SL95) and using the above values for $\langle V(\mathrm{RR})\rangle$ and $E(V-I)$, we obtain a true distance modulus of $(m-M)_0 = 17.19 \pm 0.03$ (internal) ± 0.12 (systematic) mag, or a distance of 27.4 kpc. This result is in excellent agreement with SL95.

In §4.1, we showed that the reddening in our Sgr field is indistinguishable from that in our M54 field. In SL95, we showed that the distance to stars in the Sgr field are indistinguishable from those in M54. In the remainder of this paper, we use $(m-M)_0 = 17.19$ and $E(V-I) = 0.18$ mag for both M54 and for Sgr.

6.6. Horizontal Branch Morphology

In SL95, we estimated the HB morphology of M54 to be $(B-R)/(B+V+R) = 0.87$, where B, V, and R are the numbers of blue HB stars, RR Lyrae variables, and red HB stars, respectively (Lee et al. 1994). We estimated V from the Sawyer-Hogg (1973) catalog. In the present study, we have discovered new RR Lyrae members of M54, so we can update this number.

In SL95, we also assumed that $R = 0$. Though there is evidence for a red HB in Figure 3 of SL95, we attributed this feature to the Sgr field population, rather than to M54, based on three pieces of evidence. First, the luminosity functions of our Sgr field and the Sgr field of MUSKKK had corresponding peaks at the magnitude level of the "anonymous" red HB (AHB) in the M54 field (Figure 10 of SL95), indicating that the feature was ubiquitous in Sgr. Second, the cumulative radial distribution of AHB stars (Figure 11 of SL95, also Figure 12 herein) matched those of the Sgr red clump and red giant branch, and disagreed with those of the M54 blue HB and red giant branch. Thus the AHB stars were not spatially concentrated around the cluster. Finally, we noted that the AHB was $\sim$0.15 mag fainter than the HB fiducial of M68, when the latter was fitted to the blue HB stars in M54 (Figure 6 of SL95).

Recently, this interpretation was challenged by Smith et al. (1998). They employed statistical CMD subtraction techniques on the data from SL95. In one test, they subtracted stars with $R > 200$ arcsec from the cluster center away from the stars inside that radius (see their Figure 2c). However, Trager et al. (1995) found the tidal radius of M54 to be 460 arcsec, so in this test, Smith et al. were subtracting one sample of cluster stars away from another. This biases the appearance of the HB, and explains why their subtracted CMD has so few HB stars. Clearly, this result and the conclusions derived from it are incorrect.

In a second test, Smith et al. (1998) subtracted the Sgr-field CMD of SL95 off the M54 CMD. This procedure is more nearly correct. However, there are still many Sgr red clump and red giant branch stars visible in the CMD (see their Figure 2b), indicating that too few Sgr stars were subtracted away (see §10). In our subtractions (§4.2), the anonymous red HB disappears. However, as we lower the Sgr/M54 normalization, a hint of the red HB reappears. Thus, our subtractions can not ascertain without doubt whether the anonymous red HB belongs to Sgr or to M54.

Recently, additional evidence in favor of the AHB belonging to Sgr has come from the wide-field photometry of Bellazzini et al. (1999a). In their fields "SGR12" and "SGR34", a weak AHB can be seen at $V-I < 1.0$ mag. Photometry of stars in the core of M54 are scheduled to be obtained soon with the *Hubble Space Telescope* (GO 6701). The presence or absence of red HB stars in these data should settle the question of the membership of this population. Until then, the exact HB morphology of M54 remains uncertain.

We compute two provisional values for $(B-R)/(B+V+R)$, one including and one excluding a contribution from red HB stars. From a CMD generated from stars lying between 1.0 and 4.0 arcmin from the cluster center, we count $B = 123 \pm 19$ blue HB stars, where the error enhancement over Poisson statistics reflects the uncertainty in identifying blue HB stars along the faint end of the extreme blue HB. In this same radial range, we count 44 RR Lyrae with the correct magnitude to be cluster members. Roughly 3 ± 2 of the RRL in our annulus are Sgr members (see §6), so $V = 41 \pm 7$ stars. Counting the red HB stars is complicated by the heavy contamination by foreground stars. We created a luminosity function by counting stars with $0.72 < V-I < 0.95$ in bins of $\Delta V = 0.25$ mag. The excess of stars in the bin containing the red HB ($18.05 < V < 18.30$) was found by fitting a line to the eight surrounding points in the luminosity function, and subtracting off the amount predicted at

$V = 18.20$ mag (50 stars) from the observed amount (85 stars). Thus $R = 35 \pm 10$ if the red HB is associated with M54, and $R = 0$ if it is not. From these values, we compute $(B-R)/(B+V+R) = 0.44 \pm 0.09$ in the former case, and 0.75 ± 0.04 in the latter.

The $R = 0$ case agrees reasonably well with the SL95 value of 0.87. The difference is due mostly to the newly discovered RR Lyrae presented in §5. The other case agrees quite well with the "far field subtraction" of Smith et al. (1998), but disagrees strongly with their "near field subtraction" result, which we showed above to be incorrect. Smith et al. (1998) adopted a final value by averaging the HB morphology estimates from their two subtractions, obtaining $(B-R)/(B+V+R) = 0.20$. Our much bluer HB morphology estimates place M54 back among the Galactic globulars in a plot of HB morphology *versus* [Fe/H] (see Figure 1 of Smith et al.). If $R = 0$, the cluster may be associated with the "old halo" Galactic globulars (Zinn 1996), while if the redder HB morphology value is correct, an association with the "young halo" clusters is preferred. We re-emphasize that if the red HB does belong to M54, an explanation will be required for its faintness relative to the blue HB and RRL stars. Nevertheless, it is clear that whether or not the red HB belongs to M54, the three metal-poor Sgr globulars (M54, Arp 2, and Ter 8) present no complication to the accretion picture of Galactic halo formation, contrary to the claims of Smith et al. (1998).

The unremarkable position of M54 in the HB morphology *versus* [Fe/H] diagram is also reassuring given that the main sequence turn-off age of this cluster is typical of Galactic globulars at its metallicity (§7, LS97, Marconi et al. 1998). Metallicity and age effects are sufficient to describe its HB morphology, and no additional "second" parameters need to be invoked.

7. The Age of M54 and the Sgr Star Clusters

7.1. Comparison with Cluster Fiducial Sequences

One method by which we can probe the age of M54 is by direct comparison with the fiducial sequences of other clusters. For this purpose, we have selected M92 ($[Fe/H] = -2.24$) and M5 ($[Fe/H] = -1.40$), which bracket M54 in metallicity and possess high quality $V-I$ CCD photometry available from the literature. Figure 14 shows our field-decontaminated M54 photometry from Figure 7(d). The dashed line is the fiducial sequence of M92 and the solid line is the fiducial of M5, both taken from Johnson & Bolte (1998). These have been registered to the observational HR Diagram using the V(HB), $[Fe/H]$, and $E(V-I)$ values given in Table 13, along with our adopted luminosity-metallicity relation (Chaboyer 1999).

The fiducial sequence comparison reveals that the age of M54 is comparable to those of M92 and M5, with the M54 data located between the two fiducials as one would expect based on the relative metallicities. This analysis supports the results of similar work by LS97, who used M68 and M5 as comparison clusters. M68 appears to be among the youngest metal-poor globulars (13 Gyr), while M92 is perhaps the oldest (18 Gyr), and M5 is typical of intermediate-metallicity clusters (15 Gyr; Chaboyer et al. 1996). Thus, M54 falls nicely within the age-metallicity relation defined by the Galactic globular clusters (e.g., Chaboyer et al. 1996, Figure 1). In the next section, we will apply a more precise relative age technique that is likely to shed more light on this issue.

7.2. The Magnitude of the Subgiant Branch

Since the subgiant branch (SGB) of M54 is so clearly delineated in the CMDs of Fig. 7, we have investigated the use of this feature to probe the age of M54. We should measure the V magnitude of the SGB at a specific $V-I$ color chosen so as to minimize contamination from main sequence turnoff stars and lower RGB stars scattered onto the SGB. We find that a $V-I$ color 0.21 mag bluer than $(V-I)_g$ (the color of the RGB at the level of the HB) allows the construction of the luminosity function (LF) which optimally isolates the location of the SGB. Using Gaussian fits to the cluster LF at this color yields the value of V(SGB) for M54 listed in Table 14. The quoted uncertainty represents the standard error of the mean. In a similar manner, we measured the V(SGB) values of Terzan 8 (Montegriffo et al. 1998), M92, and M5 (Johnson & Bolte 1998). In the latter two clusters (and for Ter 7), the SGBs are not populated adequately to facilitate the construction of the LFs; however, the photometry defines the SGB exceptionally well allowing us to simply compute the median magnitude of stars within $\Delta(V-I) = \pm 0.02$ around the $(V-I)_g - 0.21$ value for each cluster. Table 14 lists these measured quantities, and Figure 15 shows the Gaussian fits to the observed LFs.

For the remaining two clusters in Sgr, Arp 2 and

Ter 7, published photometry does not exist in the VI filter passbands, but it does exist in BV (Buonanno et al. 1995a, 1995b). In order to measure V(SGB) in these clusters and remain consistent with the values for the clusters measured thus far, we rely upon the $(V-I)_0$–$(B-V)_0$ transformation established by Zinn & Barnes (1996). To test the veracity of this technique, we utilize the BVI CCD photometry of M5 from Sandquist et al. (1996). The V(SGB) of M5 was measured from the VI photometry as well as the BV data after transformation to VI via the Zinn & Barnes equation. These two determinations of V(SGB) differed by only 0.01 mag, well within the errors, thereby supporting the use of the Zinn & Barnes (1996) transformation to convert the BV photometry of Arp 2 and Ter 7 to the VI system. The resultant values of V(SGB) for these clusters are also given in Table 14, and the age diagnostic ΔV_{SGB-HB} is presented for each cluster.

To convert these ΔV_{SGB-HB} values to relative ages, we rely on the Revised Yale Isochrones (RYI, Green et al. 1987) to calibrate age as a function of $[Fe/H]$ and M_V(SGB), which is valid in the range $-2.3 < [Fe/H] < -0.70$. Since we are required to measure M_V(SGB) at $(V-I)_g - 0.21$, we have used relations between $(V-I)_{o,g}$ and $[Fe/H]$ (Sarajedini 1994) to infer the theoretical value of $(V-I)_{o,g}$ which is then utilized to measure M_V(SGB) at the same point on all of the isochrones. It is important to point out that use of the Sarajedini (1994) relations between $(V-I)_{o,g}$ and $[Fe/H]$ maintains the internal consistency of our method because the abundances of M54, Arp 2, Ter 8, and Ter 7 have been determined using these same relations. In any event, we note that the exact algorithm used to set the location of the fiducial $(V-I)_{o,g}$ point for the isochrones is largely irrelevant; this is because the SGBs of isochrones with different ages are essentially parallel leaving the relative ages unchanged (the SGBs are no longer parallel for ages $\lesssim$ 6 Gyr). Table 14 shows the ages of M92, Ter 8, Arp 2, M54, M5, and Ter 7, assuming the M_V(RR)–$[Fe/H]$ relation of Chaboyer (1999). For Ter 7, which possesses a red HB, we offset the synthetic red HB models of Sarajedini et al. (1995) so that they are consistent with our adopted M_V(RR) relation. Column 6 lists the ages assuming scaled solar abundances. In summary, we find that the oldest Sgr globular clusters are similar in age to the Galactic globular clusters, while Ter 7 is some 5 to 6 Gyr younger than these clusters. In the case of M54, these results support our findings based on the fiducial sequence comparisons.

8. The Ages of the Sgr Field Populations

8.1. Isochrone Fits

We showed in Figure 6(a) the CMD of the Sgr field located 12 arcminutes north of M54. Several features of this diagram are relevant to the question of the age(s) of the Sgr field. First, the vertical extent of the dominant main sequence turnoff (MSTO) appears to span ~1 mag with at least two SGBs present, one at V ~20.4 and the other at V ~20.8. These features are also present in the field-subtracted CMDs shown in Figures 6(c) and 6(d). The presence of two or more SGBs indicates that star formation in Sgr did not occur in a single event. Bellazzini et al. (1999b) also suggested this possibility, but the poorer quality of their data near the MSTO made a definitive statement impossible; our data show the evidence for extended star formation much more clearly.

The fact that the brighter (younger) SGB extends from a turnoff with roughly the same $V-I$ color as the fainter (older) SGB indicates that the stellar population associated with the former is likely to be more metal rich than that of the latter.

With these points as the foundation, we now discuss the results of isochrone comparisons to the Sgr CMD. We make use of the theoretical isochrones published by Bertelli et al. (1994). We employ our estimates of the distance modulus and reddening from §6.5. Results in the literature suggest a metallicity range in Sgr from $[Fe/H] \sim -0.5$ to $[Fe/H] \sim -1.3$ (e.g., SL95, Ibata et al. 1997 and references therein). Bellazzini et al. (1999a, 1999b) suggest the presence of a trace population of stars at $[Fe/H] \lesssim -2$, but so few stars would be poorly represented near the MSTO, and we can not reliably identify them in our CMDs.

Figure 16(a) shows the field-subtracted Sgr CMD from Figure 6(d), together with selected Bertelli et al. (1994) isochrones for $[Fe/H] = -1.3$ dex. Figures 16(b) and 16(c) show the same observational data plotted with isochrones for $[Fe/H] = -0.7$ and –0.4 dex, respectively.

The fainter MSTO/SGB sequence, with $V(\mathrm{SGB}) \approx$ 20.8, appears to be fit very well by the isochrone with $[Fe/H] = -1.3$ and an age of 10 or 11 Gyr. The curvature of the more metal-rich isochrones around the

SGB do not match the observations, nor do the RGB colors, though we cannot rule out $[Fe/H] = -0.7$ and an age of 7 Gyr.

By contrast, none of the $[Fe/H] = -1.3$ isochrones fit the brighter MSTO/SGB sequence, with $V(\mathrm{SGB}) \approx 20.4$; the isochrone colors are too blue near the MSTO. This sequence falls between the 4 and 6 Gyr isochrones for $[Fe/H] = -0.7$, and near the 4 Gyr isochrone for $[Fe/H] = -0.4$ dex. The color of the RHB seems to favor $[Fe/H] = -0.7$ for this population. A explanatory note about the brighter sequence is required. One tends to see the SGB as the line of points which slopes upwards as it gets redder across $V - I = 0.95$, parallel to the lower sequence. However, the isochrones slope sharply downward here, indicating that the true SGB is the downward sloping line of points at $V - I = 0.90$ and $V = 20.6$ mag. The brighter line of points is probably an artifact of the subtraction — an extension of the ridge of unsubtracted stars near $(V - I, V) = (1.3, 21.6)$ and the unsubtracted clump at (0.85, 18.7) mag. The 5 Gyr isochrone for $[Fe/H] = -0.7$ follows the claimed SGB sequence nicely.

MUSKKK, Marconi et al. (1998), and Bellazzini et al. (1999a) have noted the presence of bright, blue stars in Sgr. Though it is possible that they are blue stragglers, these stars are generally interpreted as being members of a very young population in Sgr. In Figure 16, the stars with $V - I < 0.6$ and $V < 21$ do not form a tight sequence indicative of a coeval population. Many of these stars lie near the 1 Gyr, $[Fe/H] = -0.4$ isochrone, but the dispersion of stars both blue-ward and red-ward of this curve suggests that Sgr has experienced ongoing, low-level star formation over the last 2–3 Gyr. The spur of MSTO stars brighter than the upper MSTO/SGB sequence, terminating at $V \approx 20$, is consistent with a 2.5 Gyr isochrone at $[Fe/H] = -0.4$ (or somewhat younger if it is more metal-rich). We can not tell from our CMDs whether the 1.0 and 2.5 Gyr populations are discrete or represent a continuous period of star formation.

Figure 16(d) shows the field-subtracted CMD of M54 presented in Figure 7(d). The lower set of isochrones are for $[Fe/H] = -1.7$, very close to the metallicity of M54. Though the RGB color is slightly mismatched, the SGB and MSTO regions of M54 appear to be represented nicely by the isochrones with ages of 14–16 Gyr.

A spur of MSTO stars rises above the M54 MSTO in Figure 16(d), up to $V \approx 20$, very similar to the spur seen in the Sgr CMD (Figure 16c). The upper set of isochrones, for $[Fe/H] = -0.4$, demonstrate that 2.5 Gyr is a reasonable estimate for the age for this population. Hints of the SGB and upper RGB can be seen along this isochrone. The region where the SGB and RGB join is not populated, but the density of bulge stars is high here (see Figure 6b), so the elimination of a sparse population is not surprising (Bellazzini et al. 1999b). We believe that the 2.5 Gyr population seen in Figures 16(c) and 16(d) are the same population. However, since the Sgr CMD observed 12 arcmin from M54 was subtracted from the M54 data, the fact that this population appears in Figure 16(d) indicates that the relative density of the 2.5 Gyr population must be higher near M54. We shall investigate this further in §10.

To summarize this section, we find that M54 has an age of ~15 Gyr on the Bertelli et al. (1994) scale. The Sgr field is composed of several different populations, which we characterize as follows: (a) $[Fe/H] = -1.3$ and an age of 10–11 Gyr, (b) $[Fe/H] = -0.7$ and an age of 5 Gyr, and (c) a sprinkling of younger stars with $[Fe/H] \approx -0.4$ and ranging in age from perhaps 0.5 to 2.5 or 3 Gyr. Given the photometric errors and the complicated mix of stellar populations which overlap both spatially and in the CMD, it is difficult to provide definitive ages and metallicities for these populations — we give our best estimates. Furthermore, it is difficult to say whether the features in the CMD depict discrete star formation events of short duration, more extended epochs, or even continuous star formation with a variable rate. Finally, we acknowledge the difficulty in obtaining accurate absolute ages (e.g., Chaboyer et al. 1996). The relative ages determined in this section are more secure than the absolute ages. In the following section, we compare the observed Sgr sequences with those of several well-studied Galactic clusters in an effort to corroborate our isochrone-based findings.

8.2. Fiducial Comparisons

To further explore the ages and metallicities of Sgr, it is instructive to compare our statistically decontaminated CMD with fiducial sequences of open and globular clusters with similar properties. The upper panel of Figure 17 shows the fiducials of the Galactic globular clusters M92 and M5 (Johnson & Bolte 1998) and the lower panel illustrates the sequences for the open clusters M67 (Montgomery et al. 1993) and NGC 2204 (Kassis et al. 1997). The reddenings for the globular clusters come from Harris (1996) while the distances have been computed given the data in Table 13 and the RR Lyrae luminosity–metallicity relation of Chaboyer (1999). In the case of the open clusters, we have adopted the reddenings and distance moduli from the study of Sarajedini (1999). Twarog et al. (1997) indicate that NGC 2204 has a metallicity of $[Fe/H] = -0.34$ and M67 has $[Fe/H] = 0.00$.

From Figure 17, we can draw the following conclusions. The top panel indicates that the mean stellar population in Sgr has a metal abundance that is higher than that of M92 and M5 as evidenced by the mismatch in the RGB positions. The oldest population in Sgr has a main sequence turnoff that is brighter than that of M5 suggesting an age (10-12 Gyr) that is slightly younger than the canonical age (14-16 Gyr) of the Galactic globulars.

The open cluster fiducial comparisons indicate that a solar metal abundance (i.e., M67) is likely to be too high for the mean population in Sgr whereas an abundance of $[Fe/H] \approx -0.4$ to -0.3 or less is more appropriate. As for the ages, the youngest stars in Sgr are younger than NGC 2204 ($\sim$2 Gyr, Sarajedini 1999), but it is difficult to find VI photometry for a cluster with metallicity near $[Fe/H]\sim -0.4$ and an age of 1 Gyr. The intermediate age population appears to have an age close to that of M67 ($\sim$4 Gyr, Sarajedini 1999). All in all, the ages and metallicities of the Sgr stellar populations as indicated by the fiducial sequence comparisons are in line with those suggested by the isochrone comparisons.

9. The Age-Metallicity Relation of Sgr

So far in our discussion, we have determined ages and metallicities for the star clusters and field populations in Sgr. The former were studied using the difference in magnitude between the HB and the SGB calibrated with the Revised Yale Isochrones, while the latter were investigated via comparison with Bertelli et al. (1994) isochrones. The calibration of the SGB age method could not be done using the Bertelli isochrones, since the most metal-poor available are for $[Fe/H] = -1.7$, while the clusters go as low as -2.2 (M92).

In order to interpret the results for the cluster and field components of Sgr, it is necessary to place them on the same age scale. To this end, we devised a calibration between age, $[Fe/H]$, and M_V(SGB) using the isochrones of Bertelli et al. (1994). For the two clusters in our sample with metallicities in the relevant range (i.e., between $-1.7 < [Fe/H] < -0.7$), Ter 7 and M5, we find ages of 8.3 ± 1.8 Gyr and 13.6 ± 0.7 Gyr, respectively. These are $\sim$1.0 Gyr younger than those derived via the RYI. We therefore subtract 1.0 Gyr from the RYI ages in Table 14 to give ages on the Bertelli scale. Note that the age thus derived for M54, 14.7 ± 0.5 Gyr, is in excellent agreement with the age of 15 Gyr from the isochrone match shown in Figure 16(d).

Our final age estimates for the field populations and individual clusters in Sgr are listed in Table 15. The error estimates on the ages reflect the uncertainty in the age *relative* to a population of similar abundance. The error in the absolute age of a given object is about 15%. If we were to adopt a different M_V(RR)–$[Fe/H]$ relation, the entire manifold of ages would shrink or expand proportionally.

We are now in a position to synthesize the ages and metallicities of the field and cluster populations of Sgr into a coherent picture of its star formation history. Figure 18(a) shows the ages and metallicities of Sgr cluster (open symbols) and field populations (filled symbols). It is evident from the top panel of Figure 18 that there is a strong correlation between age and metallicity with the most metal-poor components of Sgr being the oldest. This behavior is qualitatively similar to that shown in Figure 8 of Montegriffo et al. (1998) and in Bellazzini et al. (1999b).

Utilizing the formalism of Binney & Merrifield (1998), we can investigate the predictions of the closed-box model of chemical evolution in the case of Sgr. Assuming that the yield of each generation of stars is constant, the variation of metallicity (Z) with time is expressed as

$$Z(t) = -p \;\; ln \frac{M_g(t)}{M_g(0)}.$$

We have assumed a constant star formation rate with time so that the ratio of gas masses $[M_g(t)/M_g(0)]$ is unity at $t = 0$ and is equal to zero at the present time ($t = 15$ Gyr). The yield is represented by p and has been adjusted to 0.005 so as to match the zero-point of the Sgr age-metallicity relation.

Figure 18(b) shows that the observed age-metallicity relation of Sgr is consistent with the predictions of a closed box continuous star formation model. This in turn argues that, if we accept the possibility that Sgr formed stars continuously, then it is likely that it did so in relative isolation with little or no infall or release of its interstellar gas.

10. Stellar Population Gradients in Sgr

SL95 and Marconi et al. (1998) noted that the more metal-rich component of Sgr is somewhat clustered around M54. Bellazzini et al. (1999a) used their wide-field observations near our Sgr field and near the field of MUSKKK ~2 degrees away to indicate that the density of Sgr stars peaks near M54. We begin our investigation of the spatial distribution of the various Sgr field population by considering the density of RGB stars in our different fields.

10.1. Density Gradients of RGB Stars

Using the CMDs in Figure 7, we have defined several loci in which to count RGB stars. The first contains the metal-poor RGB stars associated with M54[7]. For brevity, we will refer to this locus as the "z-poor box". The "z-rich box" contains the metal-rich ($[Fe/H] = -0.5$) RGB described by SL95[8]. The "mid-z box" contains the scattering of RGB stars between these boxes which SL95 characterized as an intermediate metallicity ($[Fe/H] \approx -1.3$) population in Sgr. The lack of a distinct RGB led SL95 and Bellazzini et al. (1999b) to suspect that this population exhibits a significant spread in metallicity.

We shall count stars in four different spatial regions: (1) the M54 field between 0.72 and 4.44 arcmin from the cluster center, (2) the M54 field outside 4.44 arcmin, (3) the Sgr field about 12 arcmin from the cluster center, and (4) the MUSKKK control field (the RGB stars in our control field are saturated in our photometry). The areas of these regions and the number of stars in each of the three CMD boxes are shown in Table 16.

In Figure 19, we express these counts as surface densities in units of stars per square arcminute. The error bars reflect simple Poisson statistics. We have placed the control sample at $R = 19$ arcmin for convenience of display. The top panel shows the distribution of M54 RGB stars. As expected, the density of stars in the $R = 12$ arcmin field is small and consistent with the density seen in the control field.

In panels (b) and (c), the density at $R = 12$ arcmin is significantly enhanced over the control field — Sgr exists, as shown initially by Ibata et al. (1994) and subsequently by many authors using this technique. More interestingly, the density of Sgr stars rises as we approach the center of M54. The effect is subtle for the intermediate-metallicity population; the inner point for M54 is enhanced over the density at 12 arcmin by a factor of 2.2, a 2.5σ result. The effect is much stronger for the z-rich population; the enhancement factor of 3.9 is significant at the 4.4σ level.

To conclude, we find that the density of Sgr field stars increases with increasing proximity to M54. The $[Fe/H] \approx -1.3$ population shows a mild concentration toward M54, while the metal-rich population is more strongly concentrated. If this pattern persists at larger radii, it explains why observations in more distant fields (e.g., MUSKKK, Bellazzini et al (1999a, 1999b) fail to find as many stars with $[Fe/H] = -0.5$ as did SL95 and Marconi et al. (1998) in the viscinity of M54. Furthermore, it suggests that subsequent generations of stars may have formed with greater proximity to M54.

10.2. Density Gradients of MSTO Stars

In theory, we could investigate this supposition in more detail by examining the density distributions of stars near the various main sequence turn-offs discussed in §8. In practice, the interpretation becomes more difficult for a number of reasons. The main complication is in defining boxes which sample single-aged populations. Only the most massive MSTO stars can be isolated completely; for lower masses, the main-sequences of the different populations blend together (see the isochrones in Figure 16). Thus, we can isolate stars belonging to the bright, blue MSTO stars with ages ~1 Gyr, and for the "spur" of stars above the dominant Sgr MSTOs which we estimated to be ~2.5 Gyr old. The older populations are hopelessly intermingled.

Again using the CMDs in Figure 7, we have defined two boxes in which to count MSTO stars. The first contains the ~1 Gyr population[9]. The second box contains stars in the ~2.5 Gyr old "spur" population[10]. We count stars in same four radial re-

[7] The vertices of the z-poor box are $(V-I, V) = (1.10, 17.50)$, (1.23, 16.20), (1.59, 14.92), (1.77, 15.05), (1.37, 16.26), (1.21, 17.53).

[8] The vertices of the z-rich box are $(V-I, V) = (1.29, 17.57)$, (1.65, 16.26), (2.50, 15.86), (3.20, 17.00), (1.84, 16.45), (1.38, 17.66).

[9] The vertices of the 1.0 Gyr box are $(V-I, V) = (0.32, 18.98)$, (0.63, 20.62), (0.61, 21.34), (0.30, 19.70).

[10] The vertices of the 2.5 Gyr box are $(V-I, V) = (0.65, 19.90)$, (0.74, 19.80), (0.79, 20.36), (0.63, 20.62).

gions described in §1.1, though the boundaries of the M54 annuli were shifted outward to 2.15 and 4.66 arcmin to avoid crowding effects on the faint MSTO stars. In addition, we count the MSTO stars in our control field. The areas of these regions and the number of stars in each of the two CMD boxes are shown in Table 17.

In Figure 20, we express these counts as surface densities. The error bars and control sample radii are as described for Figure 19. As for the RGB case discussed above, the two youngest populations are concentrated around M54, with density enhancements of $\sim$2 inside 6 arcmin from the cluster center. It is worth noting that these radial patterns are not artifacts of photometric detection incompleteness; if the Sgr populations were uniformly distributed across the field, our counts would tend to drop at smaller radii, where the stellar crowding due to M54 stars increases. If anything, the true density rise at small radius is underestimated in Figure 20.

Finally, we note that counts inside a box designed to isolate the 5 Gyr population discussed in §8 show a similar radial pattern. However, it is difficult to estimate the contaminating effects of the two younger populations, and we do not consider the 5 Gyr population further.

Figure 20 demonstrates that the two youngest populations are centrally concentrated around M54, as expected from the distribution of metal-rich ($[Fe/H] \approx -0.5$) RGB stars shown in Figure 19(c). It is unfortunate that we can not reliably test the radial distributions of the older populations, since they might indicate whether the spatial extent of star formation varied from one episode to the next.

10.3. M54 as the Nucleus of Sgr

Given the evidence presented above, it is clear that the Sgr field populations are concentrated around M54 with a characteristic scale of a few arcminutes. Bellazzini et al. (1999a) found evidence for clumping around M54 on the scale of several degrees, and Ibata et al. (1994) showed that M54 resides globally in the densest region of Sgr. One can easily envision that this old, massive globular cluster lies at the bottom of Sgr's gravitational potential well (the current interaction between Sgr and the Milky Way has probably disrupted the previously smooth and well-ordered stellar distributions in Sgr). Gas released from the oldest generations of evolving stars in Sgr would tend to dissipate its orbital energy and settle around M54, thus forming a preferred site for subsequent generations of metal-enriched stars. A complication to this picture is that stars from the 1 Gyr population have been detected several degrees away from M54 (MUSKKK, Bellazzini et al. 1999a). Perhaps some metal-rich stars formed during the collision of evolution-expelled gas clouds far from the bottom of the potential well.

Was the density enhancement acquired, rather than natal? Smith et al. (1998) raised the possibility that M54 might have captured stars from the Sgr field. They referred to the simulations of Bica et al. (1997), who found that a $10^5 M_\odot$ globular cluster orbiting in the Galactic bulge might capture several thousand bulge stars over the course of 1 Gyr. Smith et al. (1998) noted that the larger mass of M54 and the lower velocity dispersion of the Sgr field would increase the rate of stellar capture in M54. Detailed capture simulations designed for the case of M54/Sgr would put this possibility on a firmer footing.

In a similar vein, van den Bergh (1996) suggested that M54 may represent the merger of two globular clusters, the larger an old, metal-poor system (15 Gyr, $[Fe/H] = -1.8$) and the smaller a younger, metal-rich system like Terzan 7. van den Bergh notes that the ability for two clusters to bleed off angular momentum by ejecting stars and spinning up enhances their ability to merge, relative to the capture of a single star by a cluster. While intriguing, this scenario does not explain the fact that both the 1 Gyr and 2.5 Gyr populations are centrally concentrated, unless the merger involved two young, metal-rich clusters. This seems less likely, though multiple cluster mergers at the potential well center is considered a viable mechanism for the formation of nuclei in dwarf elliptical galaxies (Miller et al. 1998).

SL95 discussed the possibility that Sgr is a nucleated dwarf elliptical galaxy, with M54 as its nucleus. Their main objection to this conclusion was the blue color of M54 relative to the red color of the surrounding, more metal-rich field populations; the evidence then current indicated that nuclei tend to have the same color as the surrounding galaxy light (cf. Freeman 1993). More recently, cases of blue galactic nuclei have surfaced. For example, the galactic nucleus of VCC 1254 in the Virgo Cluster is bluer than its galaxy light (Durrell 1997). Given this and the improved evidence that the density of stars and mix of stellar populations in the Sgr field changes with proximity to M54, we are again encouraged to suggest that M54 is the nucleus of the Sgr dwarf elliptical galaxy.

11. Summary and Conclusions

In this paper, we describe the deep VI photometry of M54 originally presented in LS97. We also present and describe similar quality photometry of a field ~12 arcmin North of M54, whose bright stars we discussed in SL95, along with photometry of a control field on the opposite side of the Galactic bulge. We produce color-magnitude diagrams of these fields, and perform statistical subtractions to remove the foreground populations. We use these diagrams to derive the ages of the various stellar populations in Sgr and thereby outline the history of star formation in this nearby dwarf galaxy.

As a prelude to the age analysis, we determined several important cluster parameters from the variable stars in the cluster. Ours is the first CCD-based variable star survey for M54. In it, we detected and obtained light curves for 60 ab-type RR Lyrae and seven c-type RR Lyrae, along with two additional short period pulsating variables, four eclipsing binary stars, and 18 candidate long period variables (Mira, SR, or IR). Together with the previous surveys, the number of known or suspected variables in the direction of M54 now totals 120.

The mean periods of the RRL in M54 indicate that it is an Oosterhoff type-I cluster. To our surprise, we were not able to isolate RRL belonging to Sgr in the M54 field using the period-amplitude diagram, thought is is well-established that Sgr contains field RRL. We discuss this result in the context of the period-shift interpretation of Clement & Shelton (1999). We reevaluate the horizontal branch morphology of M54, which was recently questioned by Smith et al. (1998). We find $(B-R)/(B+V+R) = 0.44$ if the "anonymous RHB" described by SL95 is associated with M54, or 0.75 if it belongs to Sgr. SL95 and the data of Bellazzini et al. (1999a) provide support for the latter. In either case, our value indicates a much bluer morphology than found by Smith et al. (1998). Contrary to their claims, M54 lies among the Galactic globular clusters in the HB-morphology *versus* metallicity plot. This is consistent with the globular cluster-like age determined for M54 (Marconi et al. 1998, LS97, herein), and no additional "second parameters" need to be invoked. Also, the three old globulars in Sgr appear indistinguishable from the Galactic globulars, so the picture of Galactic halo formation via the accretion of dwarf galaxies is not violated.

Using 63 RRL in M54, we determine $\langle V(\mathrm{RR})\rangle = 18.17 \pm 0.02$ mag with an rms dispersion of 0.06 mag. This value is robust against the presence of moderate numbers of unidentified Sgr RRL in the field. From the apparent color of 29 ab-type RRL at minimum light, we determine the reddening toward M54 to be $E(B-V) = 0.14 \pm 0.03$ mag, in excellent agreement with results from the shape and color of the M54 RGB (SL95) and from the dust maps of Schlegel et al. (1998). By adopting the RR Lyrae luminosity-metallicity relation of Chaboyer (1999), we determine the true distance modulus of M54 and Sgr to be $(m-M)_0 = 17.19 \pm 0.03$ (internal) ± 0.12 (systematic) mag, or $d = 27.4$ kpc.

We use this information, together with the CMDs described above, to determine the ages of the clusters and field populations in Sgr. For the clusters, we have developed an age dating technique which measures the magnitude difference between the HB and the SGB, and used published isochrones to calibrate the magnitude difference in terms of age. We find that M54, Ter 8, and Arp 2 approximately coeval, and are as old as typical Galactic globular clusters of comparable metallicity. Comparison with the fiducial sequences of well-studied Galactic globulars, and directly with isochrones, supports this result.

For the Sgr field populations, we interpret the ages directly from isochrones which were shifted to the distance modulus and reddening values discussed above (i.e., no fitting was performed). We find that the dominant Sgr population, originally characterized by MUSKKK as 10 Gyr old with $[Fe/H] = -1.1$ dex, is actually composed of several MSTOs, or perhaps a continuum of MSTOs indicating continuous star formation with a variable rate. The two strongest MSTO signatures match the best with isochrones of 11 Gyr at $[Fe/H] = -1.3$ and 5 Gyr at $[Fe/H] = -0.7$ dex, respectively. A sparser population rises above the dominant Sgr MSTOs; we characterize this "spur" in the CMD with the parameters 2.5 Gyr and $[Fe/H] = -0.4$ dex. The extremely bright, blue stars noted by MUSKKK and others are matched reasonably with an isochrone of 1 Gyr and $[Fe/H] = -0.4$ dex. However, neither of the two younger populations presents a tight sequence in the CMD, suggesting that both might have formed over a significant period of time. Indeed, the entire Sgr CMD hints that star formation in Sgr was a continuous process, punctuated by peaks and valleys in the star formation rate, rather than by discrete episodes.

The age-metallicity relation defined by the clusters and field populations (Figure 18) is surprisingly well defined, compared with those of Montegriffo et al. (1998) and Bellazzini et al. (1999b). The main difference is the age of Arp 2; those authors place it 3–4 Gyr younger than M54 and Ter 8, whereas we find the three clusters to be nearly coeval.

We find that a simple closed-box model of chemical evolution fits our age-metallicity relation very well. Taken at face value, this suggests several things. First, Sgr must have experienced little gas-infall after its initial formation. This is consistent with the picture of dwarf galaxies as primordial building blocks of larger galaxies (e.g., Searle & Zinn 1978). Second, Sgr must have been been massive enough to retain its gas both against internal energy inputs such as supernovae and stellar winds, and against external influences like tidal interaction with the Milky Way. Sgr is a fairly luminous, hence massive, dwarf galaxy (Ibata et al. 1997), and its gas retention is qualitatively consistent with the energy injection simulations of Mac Low (1999). Ibata et al. (1997) suggested that Sgr has an especially large dark matter halo to maintain coherence during its many perigalacticon passages. Alternately, Bellazzini et al. (1999b) and others have suggested that the short-period orbit favored by Ibata and collaborators is incorrect, and that Sgr is making its first perigalacticon passage.

A full description of the star formation history of a galaxy requires an estimate of the star formation rate as a function of time, in addition to the age-metallicity relation we have derived here (e.g., Grebel 1998). Bellazzini et al. (1999b) give several estimates of this function, but their results depend heavily on their assumed age-metallicity relation. In a forthcoming paper, we will use population synthesis techniques to constrain the star formation rates of the different Sgr field populations described above directly from the density of these stars in the CMD (Liu et al. 2001).

Finally, we showed that the populations generally associated with Sgr are spatially concentrated around M54. In particular, the metal-rich, 1.0–2.5 Gyr populations are the most strongly clumped. This explains why SL95 found a higher metallicity for Sgr ($[Fe/H] = -0.5$ dex) than other studies which observed stars farther from M54; the relative densities of the different-metallicity populations vary radially, and both metallicity estimates are correct in their respective regions of Sgr. It is not currently clear what mechanism is responsible for this clumping. Does it reflect new generations of stars formed from chemically evolved gas which had settled to the bottom of the galactic potential well, or were these stars captured from the Sgr field or pre-existing clusters by the tidal influence of M54? In any case, it seems likely that the luminous globular cluster M54 is analogous to the central star clusters seen in nucleated dwarf elliptical galaxies.

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Table 1
Coordinates of Field Centers

Field	$RA_{\rm J2000}$	$Dec_{\rm J2000}$	$l^{\rm II}$	$b^{\rm II}$
M54	18:55:03	–30:28.8	5.61	–14.09
Sgr	18:55:17	–30:13.9	5.86	–14.03
Control	18:33:31	–41:10.6	–6.30	–14.35
MUSKKK Cntrl	18:33:26	–40:58.0	–6.11	–14.25

Table 2
Photometry of M54 from Shallow Frames

ID	$X_{\rm pix}$	$Y_{\rm pix}$	V	σ_V	I	σ_I	$N_{\rm obs}$
1	1240.31	18.67	14.078	0.047	14.031	0.215	3
2	1162.75	684.69	14.390	0.009	13.605	0.209	4
3	1167.17	634.97	14.396	0.004	13.621	0.008	3
4	983.93	704.52	14.493	0.005	13.751	0.012	4
5	1380.84	495.20	14.557	0.007	13.752	0.016	4
6	169.81	408.57	14.594	0.017	14.117	0.014	5
7	1385.77	914.20	14.601	0.006	13.767	0.018	4
8	1182.45	476.76	14.609	0.006	13.746	0.014	4
9	1123.60	550.27	14.616	0.010	13.507	0.006	3

Note.—This table is presented in its entirety in the electronic edition of the Astronomical Journal. A portion is shown here for guidance regarding its form and content.

Table 3
Photometry of M54 from Deep Frames

ID	$X_{\rm pix}$	$Y_{\rm pix}$	V	σ_V	I	σ_I	$N_{\rm obs}$
1	1354.35	1031.66	17.575	0.034	16.940	0.001	3
2	1462.94	1094.82	17.652	0.064	16.597	0.126	3
3	564.35	1536.87	17.713	0.004	16.889	0.014	3
4	1436.04	711.26	17.747	0.023	16.807	0.009	4
5	153.00	1451.39	17.749	0.017	16.735	0.042	3
6	514.81	1543.79	17.751	0.014	16.771	0.016	3
7	115.53	1272.89	17.759	0.009	16.865	0.007	4
8	788.36	302.76	17.776	0.031	16.698	0.018	3
9	355.20	1389.03	17.814	0.010	16.834	0.010	4

Note.—This table is presented in its entirety in the electronic edition of the Astronomical Journal. A portion is shown here for guidance regarding its form and content.

Table 4
Photometry of Sgr from Shallow Frames

$\rm ID_{SL95}$	$X_{\rm pix}$	$Y_{\rm pix}$	V	σ_V	I	σ_I	$N_{\rm obs}$
1	580.46	205.14	12.913	0.001	11.699	0.003	4
2	436.71	698.96	13.992	0.001	12.609	0.003	4
3	321.17	170.12	14.074	0.001	12.749	0.002	4
4	699.23	764.98	14.098	0.003	13.069	0.006	4
5	118.78	300.19	14.239	0.002	13.255	0.005	4
6	971.21	92.99	14.310	0.001	12.844	0.003	4
7	674.70	166.16	14.412	0.001	13.499	0.003	4
8	238.29	439.99	14.424	0.001	13.094	0.001	4
9	384.45	343.89	14.620	0.001	13.493	0.002	4

Note.—This table is presented in its entirety in the electronic edition of the Astronomical Journal. A portion is shown here for guidance regarding its form and content.

Note.—This data is identical to that in Table 2 of SL95, except that the XY coordinates have been transformed to the system defined by Figure 3.

Table 5
Photometry of Sgr from Deep Frames

ID	$X_{\rm pix}$	$Y_{\rm pix}$	V	σ_V	I	σ_I	$N_{\rm obs}$
1	1052.15	693.69	17.763	0.023	16.928	0.007	5
2	424.85	1019.85	17.809	0.007	16.922	0.010	4
3	746.79	972.15	17.837	0.016	16.917	0.008	5
4	728.44	576.48	17.846	0.018	16.953	0.007	5
5	919.97	286.07	17.847	0.005	17.001	0.009	5
6	643.08	1013.63	17.848	0.010	16.948	0.005	5
7	224.65	932.54	17.853	0.007	17.011	0.015	5
8	393.06	469.94	17.887	0.004	17.069	0.007	6
9	783.77	332.38	17.888	0.005	17.068	0.008	6

Note.—This table is presented in its entirety in the electronic edition of the Astronomical Journal. A portion is shown here for guidance regarding its form and content.

Table 6
Photometry of the Control Field

ID	$X_{\rm pix}$	$Y_{\rm pix}$	V	σ_V	I	σ_I	$N_{\rm obs}$
1	276.87	157.31	17.840	0.004	17.088	0.009	3
2	347.96	1292.79	17.924	0.015	16.994	0.004	4
3	1517.80	260.91	17.929	0.004	16.763	0.023	3
4	187.02	836.07	17.940	0.007	17.196	0.006	3
5	1148.93	1108.48	17.942	0.014	17.074	0.009	3
6	737.02	1199.04	17.947	0.036	16.749	0.009	3
7	929.88	143.74	17.960	0.008	17.188	0.009	3
8	1469.66	1287.70	17.993	0.010	17.163	0.009	4
9	610.79	1027.83	17.997	0.018	16.778	0.013	4

Note.—This table is presented in its entirety in the electronic edition of the Astronomical Journal. A portion is shown here for guidance regarding its form and content.

Table 7
Observing Runs for Variable Star Photometry

Run	Dates	Telescope	CCD	FOV	$N_{\rm V}$	$N_{\rm I}$	Symbol
1	1994/06/17–18	CTIO 0.9-m	Tek1k #1	6.8	6	6	*triangle*
2	1994/06/26–27	CTIO 0.9-m	Tek2k #3	13.6	7	7	*square*
3	1994/10/13–17	CTIO 0.9-m	Tek2k #4	13.6	7	0	*pentagon*
4	1995/06/28–29	CTIO 4.0-m	Tek2k #4	14.6	11	5	*circle*
5	1996/06/13	CTIO 0.9-m	Tek2k #3	13.6	2	2	*cross*

Table 8
Mean Photometry of Suspected Variable Stars

Star	$X_{\rm pix}$	$Y_{\rm pix}$	$\overline{V}$	$\overline{V-I}$	$\Delta V^{\rm max}_{\rm min}$	Comment[a]
V20	949.3	1055.0	18.34	0.22	0.11	not variable
V22	345.6	773.7	16.52	1.31	0.05	not variable
V27	15.6	1275.3	17.74	0.81	0.14	not variable
V53	552.8	666.9	18.13	0.86	0.37	not variable
V67	889.1	809.6	17.99	0.68	0.90	variable
V69	1364.3	42.4	17.58	0.82	1.10	variable
V72	1070.5	668.7	15.93	1.13	0.04	not variable
V73	876.1	844.9	18.08	0.66	0.62	variable
V75	912.1	851.4	17.94	0.92	1.26	variable
V79	625.9	880.9	18.10	0.47	0.67	variable
V81	756.9	912.3	17.98	0.98	0.42	variable
SV1	657.7	718.0	18.59	0.35	0.93	variable?
SV2	746.3	908.9	18.27	0.93	1.45	possible EA
SV3	803.6	828.3	17.81	1.07	1.27	variable?

[a]See Appendix for more information.

Table 9
Photometric Properties of Short Period Variable Stars

Star	$X_{\rm pix}$	$Y_{\rm pix}$	Per	Typ	$\langle V\rangle$	$\langle I\rangle$	$V_{\rm max}$	$V_{\rm min}$	$E_{\rm max}$	$N_{\rm obs}$	$N_{\rm tmp}$	*rms*	$(V-I)_{\rm min}$	$\epsilon(V-I)_{\rm min}$	$N_{\rm min}$	Note
V01	747.1	998.3	1.34769	Cep	17.257	16.589	16.795	17.776	640.34534	31	7	0.075	0.827	0.026	2	*
V02	938.3	796.5	1.09461	Cep?	17.292	16.738	16.604	17.884	641.18047	32	6	0.030	0.721	99.999	1	*
V03	1145.4	778.8	0.57536	RRab	18.201	17.525	17.632	18.529	641.62160	26	1	0.022	0.752	99.999	1	
V04	1452.0	685.6	0.48011	RRab	18.189	17.522	17.507	18.618	641.28792	26	3	0.029	0.808	0.005	3	
V05	837.8	504.6	0.57911	RRab	18.112	17.487	17.494	18.490	641.27471	32	3	0.025	0.711	0.033	3	
V06	308.0	1279.4	0.53290	RRab	18.104	17.467	17.436	18.521	641.34148	25	3	0.038	0.701	0.017	3	
V07	360.2	922.7	0.59261	RRab	18.206	17.506	17.753	18.468	641.18662	32	3	0.017	0.785	0.016	3	
V12	165.4	278.1	0.31985	RRc	16.628	16.042	16.388	16.869	641.50453	27	7	0.036	0.651	0.017	2	
V14	1210.2	1371.1	0.70489	RRab?	18.023	88.888	17.446	18.372	641.51657	9	3	0.029	1.045	99.999	1	*
V15	582.5	1101.7	0.58686	RRab	18.229	17.506	17.800	18.481	641.11748	32	5	0.054	0.787	0.009	4	
V23	1165.9	1635.8	0.51767	RRab	18.230	17.634	17.535	18.647	641.51703	20	1	0.046	0.623	99.999	1	
V28	1103.8	971.5	0.51304	RRab	18.176	17.567	17.408	18.648	641.25888	27	1	0.021	0.743	0.014	4	
V29	625.6	507.9	0.58970	RRab	18.239	17.561	17.726	18.530	641.45177	31	1	0.043	0.784	0.020	7	
V30	915.0	813.2	0.57388	RRab	18.234	17.597	17.715	18.540	641.40869	31	3	0.038	0.761	0.021	4	
V31	573.3	576.2	0.64661	RRab	18.192	17.490	17.969	18.354	641.29377	32	6	0.037	0.746	0.014	7	
V32	878.9	394.2	0.52132	RRab	18.134	17.538	17.331	18.633	641.51681	32	1	0.022	0.754	0.010	11	
V33	492.7	983.2	0.49108	RRab	18.236	17.655	17.448	18.723	641.48930	30	1	0.036	0.689	0.065	3	
V34	375.3	680.5	0.50141	RRab?	18.227	17.617	17.390	18.753	641.14426	32	1	0.105	0.747	0.013	9	*
V35	851.9	619.9	0.52672	RRab	18.213	17.651	17.492	18.649	641.58894	32	1	0.024	0.753	0.012	5	
V36	853.8	1106.0	0.59906	RRab	18.158	17.439	17.827	18.345	641.26755	32	5	0.034	0.762	0.023	3	
V37	632.5	908.9	0.62743	RRab	18.220	17.568	17.934	18.433	641.36121	31	6	0.031	0.750	0.028	3	
V38	812.4	653.0	0.61111	RRab	18.057	17.347	17.741	18.272	641.61304	31	2	0.028	0.783	0.019	5	
V39	581.0	575.5	0.59962	RRab	18.246	17.543	17.858	18.471	641.02917	32	5	0.039	0.768	0.016	5	
V40	473.3	688.9	0.58637	RRab	18.235	17.543	17.805	18.538	641.12260	31	2	0.040	0.739	0.010	4	
V41	839.9	1105.0	0.61768	RRab	18.096	17.427	17.542	18.502	641.12561	32	2	0.023	0.784	0.006	8	
V42	863.4	972.2	0.32643	RRc	18.185	17.676	17.925	18.449	641.31181	32	7	0.027	0.547	0.038	5	
V43	847.5	456.4	0.59284	RRab	18.145	17.487	17.585	18.467	641.14792	32	1	0.042	0.795	0.038	2	
V44	544.3	839.0	0.61779	RRab	18.235	17.544	17.991	18.415	640.28031	29	6	0.036	0.730	0.011	6	
V45	485.1	1087.4	0.48885	RRab	18.199	17.603	17.658	18.531	641.57369	30	5	0.038	0.729	0.011	4	
V46	639.2	726.8	0.60473	RRab	18.094	17.437	17.735	18.300	641.31119	29	5	0.033	0.751	99.999	1	
V47	948.5	740.8	0.50694	RRab?	18.174	17.553	17.642	18.476	641.55164	31	1	0.054	0.678	0.016	8	
V48	636.6	1399.1	0.68702	RRab	18.104	17.383	17.714	18.405	641.10221	27	6	0.015	0.855	0.069	6	
V49	417.6	588.6	0.33128	RRc	18.034	17.575	17.831	18.233	641.46954	32	7	0.025	0.503	0.025	3	
V50	876.2	1048.5	0.56152	RRab	18.122	17.445	17.842	18.310	641.62160	32	2	0.030	0.782	0.083	3	
V51	1201.4	1311.3	0.60545	RRab	18.205	17.484	17.810	18.434	641.55984	26	5	0.022	0.784	0.028	2	
V52	620.0	1023.2	0.64083	RRab	18.208	17.486	17.966	18.386	641.05639	28	6	0.023	0.767	0.013	8	
V54	1476.8	533.6	0.57039	RRab	18.199	17.505	17.854	18.396	641.62160	26	5	0.022	0.753	0.015	2	
V55	266.0	1159.5	0.42606	RRab	18.586	17.948	18.000	19.054	641.26159	26	4	0.028	0.744	0.034	4	
V56	427.9	44.8	0.36766	RRc	18.132	17.506	17.926	18.335	641.32895	24	7	0.021	0.703	0.018	4	
V57	673.9	1486.7	0.54663	RRab	18.189	17.578	17.512	18.593	641.29705	24	1	0.025	0.733	0.016	2	
V58	1330.4	993.8	0.54866	RRab	18.220	17.523	17.733	18.506	641.45426	26	3	0.031	0.748	0.023	2	
V59	136.2	324.0	0.59942	RRab	18.169	17.459	17.724	18.415	641.19541	27	1	0.016	0.774	99.999	1	
V60	165.9	205.9	0.57585	RRab	18.247	17.519	17.706	18.556	641.38262	27	1	0.028	0.837	0.059	2	
V61	973.1	708.6	0.60392	RRab	18.177	17.469	17.767	18.403	641.08681	28	1	0.043	0.692	0.033	2	
V62	944.8	593.0	0.61355	RRab	18.273	17.578	17.962	18.448	641.47128	32	5	0.029	0.761	0.009	7	

Table 9—*Continued*

Star	$X_{\rm pix}$	$Y_{\rm pix}$	Per	Typ	$\langle V\rangle$	$\langle I\rangle$	$V_{\rm max}$	$V_{\rm min}$	$E_{\rm max}$	$N_{\rm obs}$	$N_{\rm tmp}$	rms	$(V-I)_{\rm min}$	$\epsilon(V-I)_{\rm min}$	$N_{\rm min}$	Note
V63	422.6	727.2	0.59011	RRab?	18.265	17.618	17.850	18.501	641.16485	32	3	0.051	0.735	0.013	5	*
V65	1122.7	1363.8	0.56599	RRab	17.590	16.880	17.129	17.942	641.47331	25	4	0.040	0.762	0.011	4	
V74	413.2	1080.0	0.28892	RRc	18.119	17.654	17.877	18.391	641.48003	32	8	0.021	0.534	0.008	5	
V76	677.0	578.1	0.70702	RRab	18.083	17.390	17.739	18.344	641.42010	31	6	0.025	0.754	0.012	10	*
V77	628.8	556.7	0.57691	RRab?	18.162	17.533	17.639	18.459	641.38815	29	1	0.055	0.792	0.022	5	*
V78	704.0	981.5	0.40200	RRc?	17.953	17.370	17.764	18.138	641.40613	31	7	0.039	0.588	0.107	2	
V80	675.0	936.9	0.63089	ab?	18.141	17.442	17.875	18.284	641.51687	31	3	0.041	0.709	0.023	4	*
V82	625.7	704.4	0.58719	RRab?	18.159	17.484	17.715	18.405	641.62160	29	1	0.055	0.775	0.028	5	*
V83	86.4	513.5	0.57835	RRab	18.190	17.502	17.742	18.456	641.45735	26	5	0.038	0.694	0.050	3	
V84	613.0	788.0	0.59972	RRab	18.019	17.446	17.345	18.421	641.20839	29	1	0.044	0.816	0.070	6	
V85	519.6	763.2	0.52046	RRab?	18.135	17.499	17.635	18.428	641.27914	33	3	0.052	0.723	0.029	7	*
V86	620.3	963.3	0.58367	RRab	18.147	17.487	17.634	18.519	641.18793	31	2	0.047	0.764	0.012	4	
V87	667.0	1045.0	0.55512	RRab	18.211	17.545	17.722	18.497	641.36791	32	3	0.048	0.799	0.020	3	
V88	1148.1	459.5	0.48578	RRab?	18.187	17.559	17.581	18.540	640.56880	20	1	0.075	0.699	0.022	6	*
V89	1218.1	591.6	0.61083	RRab	18.255	17.563	17.730	18.553	641.51654	21	1	0.037	0.769	0.005	2	
V90	1086.8	950.4	0.61307	RRab	18.165	17.460	17.812	18.426	641.29177	30	4	0.022	0.786	0.021	3	
V91	1261.4	1522.5	0.40978	EA?	17.373	16.907	17.137	19.787	641.51670	13	10	0.047	1.103	99.999	1	
V92	744.9	887.4	0.48470	RRab	17.879	17.117	17.240	18.272	641.62160	25	3	0.062	0.780	0.233	5	
V93	1157.9	771.3	0.55809	RRab	17.475	16.746	17.077	17.692	641.17122	25	1	0.017	0.811	0.002	3	
V94	1299.0	1022.3	0.64440	RRab	18.230	17.467	18.024	18.375	641.39542	26	4	0.023	0.804	0.021	3	
V95	559.8	745.3	0.55838	RRab	18.007	17.212	17.499	18.293	640.25483	24	1	0.034	0.860	0.037	4	
V96	-57.4	1444.5	0.55511	RRab	18.206	17.487	17.692	18.496	641.22414	20	1	0.039	0.792	0.047	2	
V97	835.1	875.0	0.33673	RRc	18.131	17.640	17.905	18.357	640.53505	29	7	0.042	0.591	0.031	7	
V98	662.3	1027.8	0.62435	RRab	18.177	17.470	17.876	18.403	641.14585	30	6	0.031	0.758	0.017	5	
V99	1153.6	626.0	0.34769	EW?	18.337	17.546	18.208	18.584	641.29129	25	9	0.043	0.800	0.028	3	*
V100	818.9	772.8	0.96473	RRab?	18.223	17.590	17.787	18.566	640.59600	16	6	0.074	0.748	0.072	3	*

Table 10

Mean Photometry of Long Period Variable Stars

Star	$X_{\rm pix}$	$Y_{\rm pix}$	$\overline{V}$	$\overline{V-I}$	$\Delta V_{\rm min}^{\rm max}$	Comment
V25	897.2	657.7	15.39	1.81	0.83	–
V66	1217.5	1338.6	16.24	3.20	0.27	–
V71	970.4	734.0	14.95	1.63	0.28	–
V101	716.6	212.2	16.54	2.79	0.70	–
V102	103.9	1133.4	16.95	3.02	0.34	–
V103	712.3	621.8	17.74	4.65	1.12	–
V104	1173.6	677.2	15.98	2.52	0.53	–
V105	817.0	531.7	16.08	2.54	0.27	–
V106	118.0	758.7	15.56	2.30	0.22	–
V107	720.1	854.2	16.22	1.40	1.25	crowded, probable LPV
V108	1196.0	1356.0	16.12	2.70	0.20	low amplitude, possible LPV
V109	1009.4	1114.5	15.27	1.56	0.12	low amplitude, possible LPV
V110	798.9	1138.7	15.19	1.69	0.12	low amplitude, possible LPV
V111	773.4	740.2	15.39	1.71	0.67	crowded, probable LPV
V112	754.6	766.4	15.33	1.81	0.60	crowded, probable LPV
V113	43.7	524.4	16.12	2.47	0.12	low amplitude, possible LPV
V114	671.6	471.4	15.35	1.65	0.07	low amplitude, possible LPV
V115	791.8	691.2	15.17	1.72	0.09	low amplitude, possible LPV

Table 11

Mean Photometry of Faint Variable Stars

Star	$X_{\rm pix}$[a]	$Y_{\rm pix}$[a]	$\overline{V}$	$\overline{V-I}$	$\Delta V_{\rm min}^{\rm max}$	Comment
V116	1169.7	989.3	19.43	0.42	0.59	binary
V117	1141.7	1313.4	19.03	0.94	0.31	binary
Sgr1	378.9	555.5	18.25	0.88	0.46	RR Lyrae
Sgr2	652.0	858.8	20.38	0.62	1.31	binary

[a]Coordinates for V116 and V117 refer to Figure 1, while coordinates for Sgr1 and Sgr2 refer to Figure 3.

Table 12
Time-Series Photometry for All Variable Stars

Star	Time	Phase	V	σ_V	I	σ_I	Code[a]
V01	520.88660	0.289	17.321	0.010	16.570	0.013	33
V01	520.89368	0.294	17.335	0.009	16.623	0.014	33
V01	520.89972	0.298	17.361	0.011	16.607	0.013	30
V01	521.66339	0.865	17.119	0.012	16.595	0.012	33
V01	521.67090	0.871	17.114	0.011	16.522	0.009	33
V01	521.67688	0.875	17.056	0.012	16.527	0.013	33
V01	529.63379	0.779	17.453	0.014	16.745	0.020	40

Note.—This table is presented in its entirety in the electronic edition of the Astronomical Journal. A portion is shown here for guidance regarding its form and content.

[a]Code corresponding to the symbols in Figures 8 and 9. See footnote 6 for an explanation.

Table 13
Observed Cluster Parameters

Cluster	V(HB)	$[Fe/H]$	$E(V-I)$
M92	15.10 ± 0.03[a]	-2.24 ± 0.08[b]	0.03[c]
M54	18.17 ± 0.02[d]	-1.79 ± 0.08[d]	0.18[d]
M5	15.09 ± 0.02[e]	-1.40 ± 0.06[b]	0.04[c]

[a]Carney et al. (1992).

[b]Zinn & West (1984).

[c]Harris (1996).

[d]This paper.

[e]Sandquist et al. (1992).

Table 14

Cluster Ages from the SGB

Cluster	$[Fe/H]$	V(HB)	V(SGB)	ΔV	Age
M92	-2.24 ± 0.08[a]	15.10 ± 0.03[c]	17.89 ± 0.03	2.79 ± 0.04	14.5 ± 0.5
Ter 8	-2.00 ± 0.10[a]	17.95 ± 0.05[d]	20.77 ± 0.02	2.82 ± 0.05	15.5 ± 0.8
Arp 2	-1.84 ± 0.09[a]	18.23 ± 0.05[e]	20.94 ± 0.03	2.71 ± 0.06	14.1 ± 0.9
M54	-1.79 ± 0.08[b]	18.17 ± 0.02[b]	20.99 ± 0.02	2.82 ± 0.03	15.7 ± 0.5
M5	-1.40 ± 0.06[a]	15.09 ± 0.02[f]	17.87 ± 0.03	2.78 ± 0.04	14.5 ± 0.7
Ter 7	-0.82 ± 0.15[a]	17.76 ± 0.07[e]	20.44 ± 0.04	2.68 ± 0.08	9.3 ± 1.8

[a]Zinn & West (1984).

[b]This paper.

[c]Carney et al. (1992).

[d]Montegriffo et al. (1998).

[e]Sarajedini & Layden (1997).

[f]Sandquist et al. (1992).

Table 15

Final Metallicities and Ages

Population	$[Fe/H]$	Age
Old Field	-1.3 ± 0.2	10.5 ± 1.5
Intermed Field	-0.7 ± 0.2	5.0 ± 1.0
Young Field	-0.4 ± 0.3	0.5 – 3.0
Ter 8	-2.00 ± 0.10	14.5 ± 0.8
Arp2	-1.84 ± 0.09	13.1 ± 0.9
M54	-1.79 ± 0.08	14.7 ± 0.5
Ter 7	-0.82 ± 0.15	8.3 ± 1.8

Table 16
Red Giant Star Counts

Field	Range	Area	Z-Poor	Mid-Z	Z-Rich
M54	100-620 pix	60.4	227	33	43
M54	>620 pix	61.4	42	21	26
Sgr	all	67.9	11	17	12
MUSKKK	all	225.6	30	6	6

Table 17
MSTO Star Counts

Field	Range	Area	1.0 Gyr	2.5 Gyr
M54	300–650 pix	53.7	64	189
M54	>650 pix	55.2	35	102
Sgr	all	67.9	44	88
Control	all	131.5	4	91
MUSKKK	all	225.6	10	106

Figure Captions

Fig. 1.— A 30 second V-band exposure of M54. The field is 11.1 arcmin on a side. North is to the right and East is up. The coordinate system employed in Tables 2 and 3 correspond to this figure, with (X, Y) = (1, 1) at the lower-left and (1551, 1551) at the upper-right. The cluster center is at (X, Y) = (726, 806) pixels. Three stars from Table 2 are marked with their ID numbers (37, 46, 49), and three stars from Table 3 are also marked (15, 51, 70). A FITS-format image of this figure is available on request from ACL.

Fig. 2.— Differences between the shallow M54 4.0-m photometry and the M54 photometry of SL95, in the sense (4.0-m – SL95). Panels (a) and (b) show the magnitude and color differences as a function of V magnitude. Panels (c) and (d) show the differences as a function of $V-I$ color. Panels (e)–(h) show the differences as a function of X and Y position in the 4.0-m coordinate system. All stars appearing in both photometry sets were used, regardless of the size of their estimated errors.

Fig. 3.— A 600 second V-band exposure of our Sgr field. The image is 8.2 arcmin on a side. North is to the right and East is up. The coordinate system employed in Tables 4 and 5 correspond to this figure, with (X, Y) = (1, 1) at the lower-left and (1150, 1150) at the upper-right. Three stars from Table 4 are marked with their ID numbers (20, 49, 68), and three stars from Table 5 are also marked (5, 12, 86). A FITS-format image of this figure is available on request from ACL.

Fig. 4.— Differences between the present Sgr 4.0-m photometry and the Sgr photometry of SL95, in the sense (4.0-m – SL95). The panels are as for Figure 2.

Fig. 5.— The CMDs of (a) the Sgr field, and (b) the M54 field. Deep and shallow photometry sets have been spliced together as described in the text. Only stars with errors in V and $V-I$ less than 0.10 and 0.14, respectively are shown.

Fig. 6.— The CMDs from (a) the Sgr field, and (b) our control field. Only stars with errors in V and $V-I$ less than 0.10 and 0.14, respectively, are shown. In (c), a subset of the stars from (b) have been subtracted statistically from those in (a), resulting in a clearer view of the Sgr sequences. In (d), a subset of the control sample of MUSKKK has been subtracted from the stars in (a), providing a complementary view of Sgr.

Fig. 7.— The CMDs from (a) all the stars in the M54 field, (b) the Sgr field, and (c) stars in the M54 field with radii between 300 and 600 pixels. Only stars with errors in V and $V-I$ less than 0.10 and 0.14, respectively, are shown. In (d), the stars from (b) have been subtracted statistically from those in (c), resulting in a clearer view of the M54 sequences alone – note that some traces of the youngest Sgr population remain.

Fig. 8.— V-band light curves for 71 short-period variables, listed by variable star designation. The points represent observations and the curve represents the best-fit template. The point symbols are described in footnote 6. In each panel, the magnitude range is 1.7 mag and the minor tick marks are separated by 0.1 mag, except in the case of V91.

Fig. 9.— Magnitude as a function of time for the long period variable V25. The time axis is broken into the five observing runs as marked. The top set of plots are for V magnitude, and the bottom set are for I magnitude.

Fig. 10.— Finder charts for each variable star in our field of view, listed by variable star designation. Each finder is 20 arcsec on a side, and is oriented with North to the top and East to the left.

Fig. 11.— The CMD from stars in Table 2 with errors $\sigma_V < 0.10$ and $\sigma_{V-I} < 0.14$, and located more than 150 pix from the cluster center. The mean magnitudes and colors of the variable stars are marked as follows: ab-type RR Lyrae (*circles*), c-type RR Lyrae (*triangles*), Cepheids (*stars*), eclipsing binaries (*squares*). For the RR Lyrae, filled and open symbols indicate stars with template fits having rms values less than and greater than 0.033 mag, respectively.

Fig. 12.— The cumulative radial distributions of blue HB stars (BHB), RR Lyrae stars (RRL), red clump stars (RC), and "anonymous red HB" (AHB) stars in our field of view. The AHB was defined as all stars with $18.17 < V < 18.33$ and $0.75 < V-I < 0.90$. SL95 showed that the BHB stars belong to M54, while the RC stars belong to Sgr.

Fig. 13.— The V-band amplitudes of our RR Lyrae stars are plotted as a function of their logarithmic periods. Symbols are as for Figure 11, except that squares indicate the four field RRL. Symbols overstruck with a *cross* lie more than 500 pix from the cluster center. The solid lines show the period-amplitude relations for M3 and M9 derived by Clement & Shelton (1999), and the dotted line is the M3 relation shifted to the position expected for RRab in a population with $[Fe/H] \approx -1.1$ (Sandage 1982).

Fig. 14.— The field-subtracted CMD of M54 from Figure 7d. The curves are the fiducial sequences of M5 ($[Fe/H] = -1.40$, solid curve) and M92 ($[Fe/H] = -2.24$, dashed curve) from Johnson & Bolte (1998), shifted to M54 as described in the text.

Fig. 15.— The observed luminosity functions at $(V-I)_g - 0.21$ mag for the indicated clusters, expressed as a difference from V(HB). The curves are Gaussian fits to the SGB peaks; the maximum of each curve is used to estimate the age of the cluster.

Fig. 16.— (a) The field-subtracted Sgr CMD from Figure 6d. The curves are Bertelli et al. (1994) isochrones for $[Fe/H] = -1.3$ dex and ages (from left to right) 0.004, 1.0, 2.0, 4.0, 6.3, 7.9, 10.0, 12.0, and 13.8 Gyr. (b) The same as in panel (a) but the isochrones are for $[Fe/H] = -0.7$ dex. (c) The same as (a) but the isochrones are for $[Fe/H] = -0.4$ dex. (d) The field-subtracted M54 CMD from Figure 7d. The lower set of isochrones are for $[Fe/H] = -1.7$ and ages 10.0, 12.0, 13.8, and 15.8 Gyr. The upper set of isochrones are for $[Fe/H] = -0.4$ and ages 2.0, 2.5, and 3.2 Gyr.

Fig. 17.— The field-subtracted CMD of Sgr from Figure 6d. (a) The curves are the fiducial sequences of M5 ($[Fe/H] = -1.40$, solid curve) and M92 ($[Fe/H] = -2.24$, dashed curve) from Johnson & Bolte (1998). (b) The curves are the fiducials from M67 ($[Fe/H] = 0.00$, solid curve, Montgomery et al. 1993) and NGC 2204 ($[Fe/H] = -0.34$, dashed curve, Kassis et al. 1997). The fiducials were shifted to Sgr as described in the text.

Fig. 18.— The age-metallicity relation for Sgr. The values for the clusters (circles) and field components (squares) are taken from Table 15. (b) The data are repeated with the age-metallicity relation predicted by the closed-box chemical enrichment model described in §9.

Fig. 19.— Projected stellar density of RGB stars as a function of radial distance from M54 in arcmin. (a) Stars in the metal-poor box defined by the RGB of M54, (b) stars in the intermediate-metallicity box, $[Fe/H] \approx -1.3$, and (c) stars in the metal-rich box, $[Fe/H] \approx -0.5$ dex. The MUSKKK control sample point is located at $R = 19$ for convenience.

Fig. 20.— Stellar density of MSTO stars as a function of radial distance from M54 in arcmin. (a) Stars in the "1 Gyr population" box defined by the plume of young, blue Sgr stars, and (b) stars in the "2.5 Gyr population" described in §9. The control fields are placed at $R \approx 19$ (the open circle represents the MUSKKK field).

A. Details on Individual Variable Stars

V1 is listed in SH73 as a Cepheid with P=1.34956 days. Our period, P=1.34769 days, is close to this value, and the mean magnitude is about 0.9 mag brighter than the cluster RR Lyrae. These properties are consistent with the Period-Luminosity relation for a Population II Cepheid pulsating in the first-overtone (Nemec et al. 1994). The star's position 3.2 arcmin from the cluster center suggests it is a member of M54.

V2 is listed in SH73 with P=0.5111 days. Our period search found no strong minima below P=0.8 days, and produced an excellent light curve at P=1.09461. The light curve properties are similar to the Anomalous Cepheids in the Sextans dwarf spheroidal galaxy found by Mateo et al. (1995b). The mean magnitude of this star is about 0.9 mag brighter than the cluster RR Lyrae, in rough agreement with the Period-Luminosity relation for an Anomalous Cepheid pulsating in the fundamental mode (Nemec et al. 1994). The star's position 3.5 arcmin from the cluster center suggests it is a member of M54.

V14 appears to be variable in our data set, but is present in only a few of our frames. We folded the existing data by a series of periods around the SH73 value of P=0.6892, and obtained a best fit at P=0.70489 days. The small number of observations makes the period and mean photometry derived for this star quite uncertain. The XY position in SH73 is in error; the variable star marked in RN59 is correct.

V20 appears not to be variable, based on the star marked on the RN59 finder.

V21 could not be identified on our images. The RN59 finder chart position disagrees with the *XY* position listed in SH73. Five candidate stars in the regions in question were checked for variability, but no significant variations were detected. A note in SH73 questions the variability of this star; we support this conclusion.

V22 appears not to be variable, based on the star marked on the RN59 finder.

V26 could not be identified on our images. The RN59 finder chart position disagrees with the *XY* position listed in SH73. Four candidate stars in the regions in question were checked for variability, but nothing significant was detected.

V27 appears not to be variable, based on the star marked on the RN59 finder. However, the stellar image is badly blended. SH73 lists this star with a single magnitude (rather than with a maximum and minimum), and the note "med" appears in the magnitude comments.

V34 light curve has large scatter; period and mean photometry are consequently uncertain.

V47: light curve has large scatter; period and mean photometry are consequently uncertain.

V53 appears not to be variable, though it was identified positively with the star marked on the RN59 finder.

V63: light curve has large scatter; period and mean photometry are consequently uncertain. Could also be an RRc with P=0.39339 days.

V67 was identified positively from the RN59 finder chart, and appears to have real variations, but we could not determine a period for this star.

V69 is blended our our images, and the resulting photometric confusion makes it difficult to tell whether this star is truly variable.

V72 does not vary significantly in our data set. The identification was made using the RN59 finder. SH73 lists it as a possible eclipsing binary; there may have been no eclipses during our observations.

V73 could not be identified on our images. The RN59 finder chart position disagreed with the *XY* position listed in SH73. Five candidate stars in the regions in question were checked for variability. Possible variations were detected in the star marked in Figure 10, but no acceptable periods were found.

V75 could not be confirmed as a variable in our data set, since its image is badly blended with that of another star.

V76 is marked on the RN59 finder as a close pair of stars, neither of which appears to be variable. However, we detected an ab-type RR Lyrae 3 arcmin East of the marked star, and suspect that the RN59 finder is in error (indeed, RN59 classified V76 as an RR Lyrae). We have assigned this RR Lyrae to V76.

V77: light curve has large scatter; period and mean photometry are consequently uncertain.

V79 could not be identified positively on the RN59 finder. Two candidate stars in the region in question were checked for variability. Definite variations were detected in the star marked in Figure 10, but no acceptable periods were found.

V80: crowded! Identification from RN59 finder uncertain, but variability is certain in the star marked on Figure 10, and a reasonable period was determined, despite the relatively large scatter in the resulting light curve. This star may exhibit the Blazhko effect.

V81: crowded! Identification from RN59 finder not possible. Five candidate stars in the region in question were checked for variability. Possible variations were detected in the star marked in Figure 10, but no acceptable periods were found. No magnitude or period information were listed in SH73

V82: light curve has large scatter; period and mean photometry are consequently uncertain.

V88: light curve has large scatter; period and mean photometry are consequently uncertain.

V99 is a possible W UMa contact binary. The light curve has large scatter; period and mean photometry are consequently uncertain.

V100: light curve has large scatter and relatively poor phase coverage; period and mean photometry are consequently uncertain.

SV1 is a new suspected variable. Could not determine period.

SV2 is a new suspected variable. Obtained a reasonable Algol light curve at P=2.11765 days, but too few data points to be certain.

SV3 is a new suspected variable. Could not determine period.

SV4 is a new suspected variable. Obtained a reasonable Algol light curve at P=0.40978 days, but too few data points to be certain.