Thermodynamics and Flexure of Beams as applied to a Racing Kart

Matthew Hodek
Bowling Green State University

Adviser:
Dr. Robert Boughton
Bowling Green State University

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1 Introduction

1.1 Quantum Racing

This study is done in relation to the Society of Physics Students race team called Quantum Racing. This team was formed as an entrant in the inaugural Grand Prix of BGSU in the spring semester of 2006. The team not only participates for the enjoyment of racing, but to learn physics in a real life setting. The kart referred to in this study is the kart fielded by Quantum Racing as is pictured in figure 1.

Figure 1: The Quantum Racing kart driven by Jen Bradley in the 2006 Grand Prix of BGSU.

It has a base racing chassis manufactured by Haase with a custom role cage and safety system built in house at the physics department.

1.2 Main Project Goals

In this study the areas of the thermodynamics of a small internal combustion engine and the chassis dynamics of a racing kart are explored. The main goals of this are to get a better understanding of the components of the Quantum Racing #42 kart and use this understanding to improve the performance of the kart. However this study is not only useful for improving the performance of the kart, but it is also important in other ways as well. Getting students involved with learning the physics behind the kart is a great way of giving them hands on research experience. This study is breaking new ground and setting the stage for further studies to come involving physics students of all levels.

2 Thermodynamics of an IC Engine

2.1 Importance of a Thermodynamic Understanding

To improve the performance of any powered vehicle, the first obvious place to go to is the power source. The Quantum Racing kart is powered by a Honda GX160 single cylinder overhead valved general purpose engine running on E85 fuel. This particular engine is manufactured with the intended fuel of gasoline, and it was converted to run on E85. The main focus of this area of study is to understand how running the engine on E85 fuel effects the engine's performance.

2.2 Basic Thermodynamic Model (Air Cycle

2.2.1 Assumptions

The simplest thermodynamic model of an engine, also known as an air cycle, makes many assumptions. The first thing is to assume that the engine has a volumetric efficiency of 1. This means that the engine is essentially a perfect fluid pump drawing in a full volume of air and fuel (also known as the charge) equal to the displacement of the cylinder. At slow RPMs this is not a bad assumption, but as the speeds increase this efficiency of being able to draw in a full charge become a greater factor. Also the volume of fuel is rather small in comparison to the air in the charge, so its volume is ignored. Another assumption is that the charge is completely burnt, releasing all of its energy. There are also the issues of thermodynamical efficiencies resulting for complex chemical processes, and mechanical efficiencies resulting from the physical design of the engine. All of these different efficiencies are grouped together into a total efficiency.

2.2.2 Model Theory and Predictions

The calculations dealing with the simple thermodynamic model is based around two related equations of the horsepower (HP) and the torque. The HP is calculated in theory by calculating how much fuel is burnt per minute and is expressed in the equation

$$HP = \frac{J}{K_p} \dot{M}_a f Q_c \eta \tag{1}$$

Where J/K_p is a dimensional constant (0.707 BTU/sec in the foot/pound/sec unit system), $\dot{M}_a$ is the mass of air per unit time, f is the fuel to air ratio, Q_c is the heat of combustion per unit mass of fuel, and η is the total efficiency. In order to create a HP vs RPM graph that equation needs to be modified by substituting in a RPM dependent function for the mass of air. By assuming that air is an ideal gas, the equation can be modified to

$$HP = \frac{J}{K_p} \frac{VP}{\frac{R_{air}}{m} T(2 \times 60)} f Q_c \eta \tag{2}$$

with the cylinder displacement V, air pressure P, R_{air}/m is the universal air constant over the molecular mas of air,and T is the absolute temperature. The factor of 2×60 takes into account that this is a four cycle engine that fires on every other revolution and that to convert the RPM into revolutions per second. The torque in general is the derivative of HP. On a HP vs RPM graph the torque for a given RPM can be calculated with the HP at that RPM by the equation

$$Torque = \frac{HP \times 5252}{RPM} \quad (3)$$

Using these simple equations and the values listed in the table, a graph of the Hp and Torque vs RPM can be produced.

variable	value
K_p/J	0.707 BTU/sec
B_{air}/m	53.3 ft lbf/degF
V	0.005729 ft^3
T	530degR
P	212 lbf/ft^2
f	gasoline .6
	E85 .12
Q_c	gasoline 20460 Btu/lbm
	E85 13932 Btu/lbm

Table 1: Equation variable values

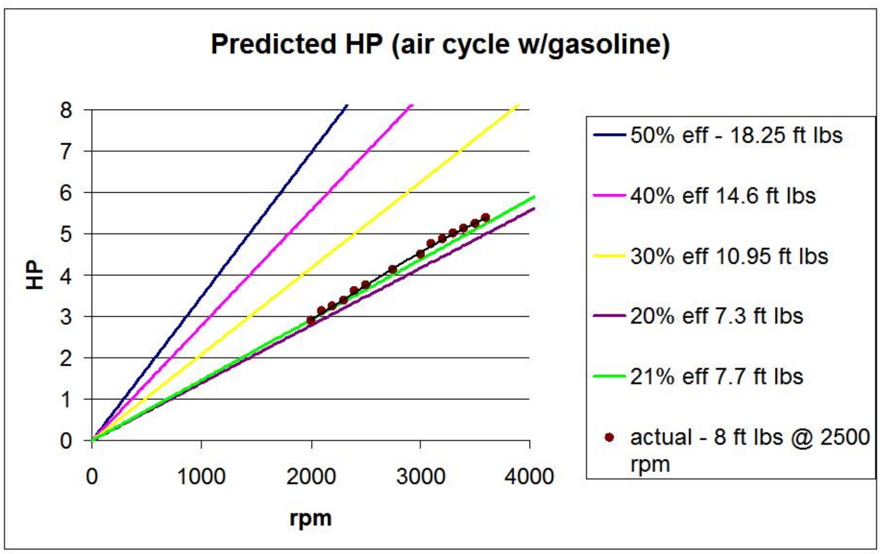

Figure 2: Air Cycle Predictions

In plot 2 there are a set of data points obtained from the engine manufacture as measured HP values. This indicates that the total efficiency of the engine is approximately 22%. [1]

2.3 Problems With the Air Cycle and more Advanced Models

Obviously the air cycle is a very basic approximation. The first indication of an issue is that it produces a linear HP plot. In a measured HP curve there is a maximum value and the plot curves over as seen in 3. This is due to the volumetric efficiency playing a larger role at higher RPMs.

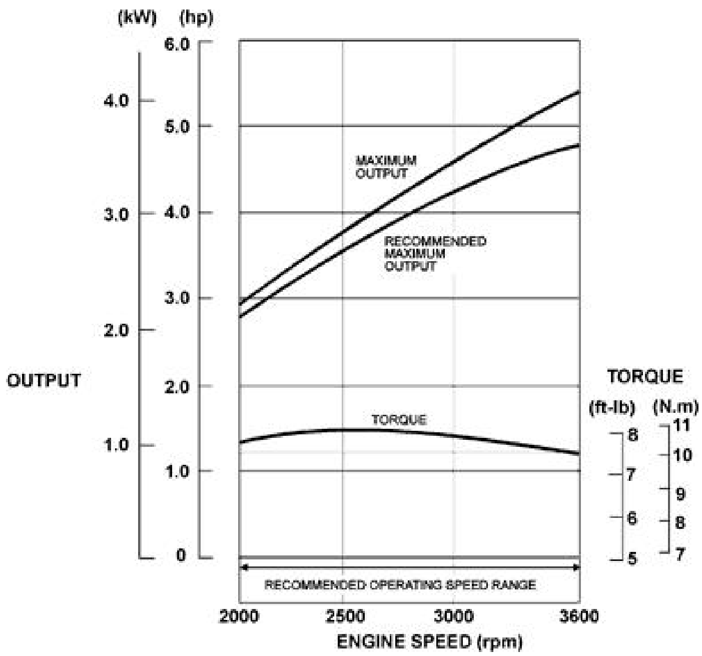

Figure 3: Manufacturer's Measured HP Curve

The air cycle also assumes a very simplistic view of the chemical process of combustion. More advanced models known as the fuel-air cycle or the actual cycle take into account those complexities such as chemical equilibriums and traveling flame fronts. Unfortunately, incorporating those issues into a working model is not an easy process. Either the model can use large amounts of empirical data, or it can use rather complex mathematical formulas to compute those values. Either of these methods are beyond the scope of this initial study, and may be considered in the future.

2.4 Basic Data from Kart

2.4.1 Data Gathering and Analysis

In any research area theoretical calculations are good, but they mean nothing without experimental confirmations. Unfortunately access to a small engine dynamometer was unavailable. Instead the on-board computer system installed on the kart itself was used. The computer is the Alfano Astro

Chronometric System that measure the head temperature, lap times, engine RPM, wheel speed, and latitudinal and longitudinal accelerations at a rate of ten hertz. The torque produced by the engine is measured in relation to the longitudinal acceleration. This can be done by taking into account that the acceleration is due to a force between the tired and the track surface, and that the force is related to the torque of the engine through the gearing. This whole relationship can be expressed in the equation

$$Torque = a_{long} M_{kart} R_{tire} \frac{teeth_{clutch}}{teeth_{axle}} \tag{4}$$

It might seem that the torque transfer through the clutch sprocket to the axle sprocket would be dependent on the radius of the sprockets and not the teeth. However, as with all karts, a standardized chain is being used which mean that the teeth per inch on all sprockets for the chain are the same. This in turn means that the ratio of the sprockets' radii is exactly proportional to the ratio of the teeth. By using the same equation used previously to go between the HP and torque, the measured HP can be determined. Figure 4 shows the highest measured torque and HP for each RPM that was read. It is apparent that there is some what of a curve present but there is too much noise for it to

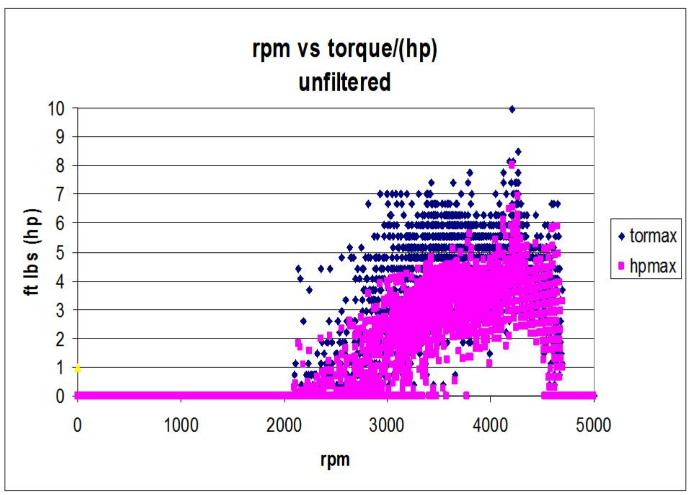

Figure 4: Measured HP/torque

be useful. To deal with this the data was put through a 'binning' program that separates the RPMs into a range and then picks the greatest torque and HP from within each bin. The best results shown in figure 5 were found by using a bin that had a 20 RPM width (referred to on the diagram as delta). This measured data can be compared to the calculated data. This time the calculated data was produced using the values for E85 as the kart was run on that fuel. These results are shown in figure 6.

2.4.2 Issues with Current Data Recording Methods

While the calculated and the measured data is in at lest decent agreement, there are some issues with the measured data. The first is dealing with the fact that while the kart is accelerating it may

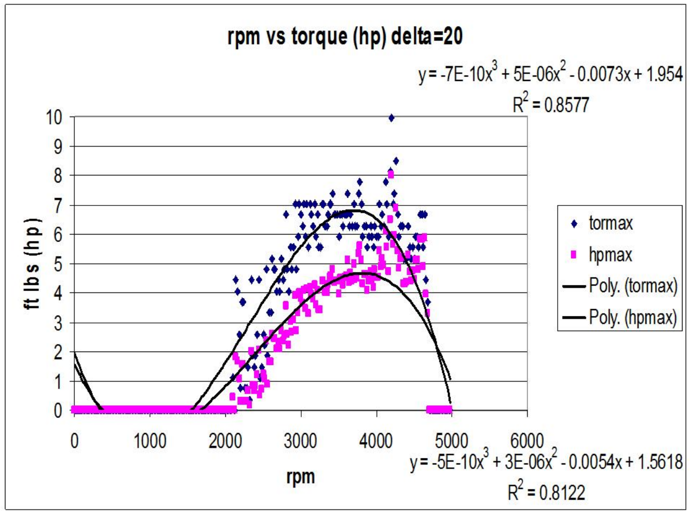

Figure 5: Measured HP/torque curves after the binning process.

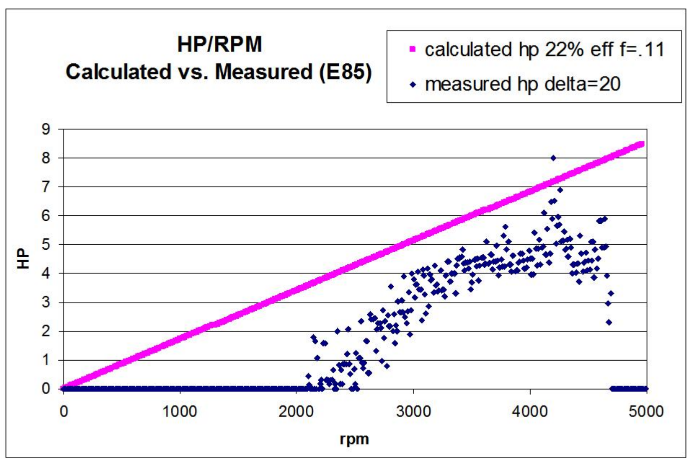

Figure 6: A comparison of the calculated and measured HP/torque curves using E85 fuel.

or may not be under maximum load. This is simply due to the fact that these measurements were taken under race conditions where such things are not controlled. This effect is very apparent in the figure **??** to explain why there is such a difference in HP/toque values for relatively close RPMs. The second problem also stems from the fact that the measured data is recorded under race conditions instead of on a dynamometer, and it has to do with the clutch installed on the kart. It is necessary to use a clutch to allow the engine to idle while that kart is sitting still. This effects the measured data in that the computer won't record an acceleration, and hence any HP/toque data, until the engine RPM is above the value needed to engage the clutch. In any of the three figures involving the measured data, it can be seen that there is no data recorded before a little past 2000 RPM where the clutch is set. This offset changes how the engine reacts to a load, and therefor changes the curve in general.

3 Chassis Dynamics

3.1 Importance of Understanding the Chassis

Once the power of the kart is understood and is tuned, the next thing to look at is how the kart handles in the corners. Many people say that how the kart handles and how the driver takes the corners makes the biggest difference in how the kart performs. On most racing vehicles this involves understanding the suspension system, but karts do not have a suspension. Instead they rely on the chassis itself to flex and move in a way that is helpful in the corners. To understand the chassis on a whole is a rather large task due to the fact that the chassis is a collection of tube elements that all interact with each other as they flex. To begin to understand this, it is first necessary to understand how a beam element flexes on its own.

3.2 Flexure of Beams

The first concept to understand in the flexure of beams is that of the bending moment. The bending moment of a beam at a certain point is defined as the sum of the moments, or torques, to either the left or right of the point with respect to that point. These can be due to either torques themselves or from a force at a distance. The decision of whether to use the left or right forces is merely a matter of convention and should not effect the outcome if the calculations are consistent. Figure 7 shows the simple case that is being analyzed here. The a load W is placed a distance D from the

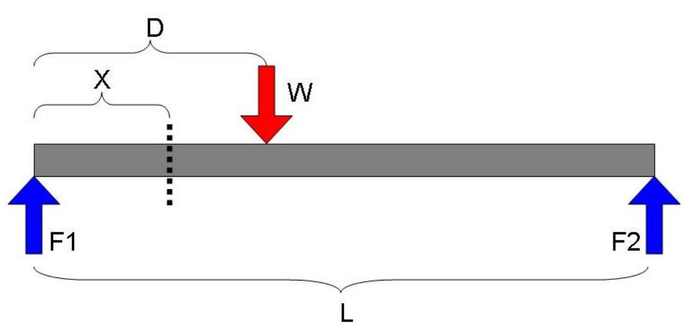

Figure 7: Force diagram for the flexure of a beam.

left end, and two forces F1 and F2 are on the ends of a beam of length L. The position along the beam is denoted by the variable x. With this situation the moment as a function of position needs to be separated into two sections, one $x \leq D$ and one $x \geq D$.These two moment equations are equal valued at D and are shown in figure 8.

$$M = F2(L - x) - W(D - x) \quad x \leq D \tag{5}$$

$$M = F2(L - x) \quad x \geq D \tag{6}$$

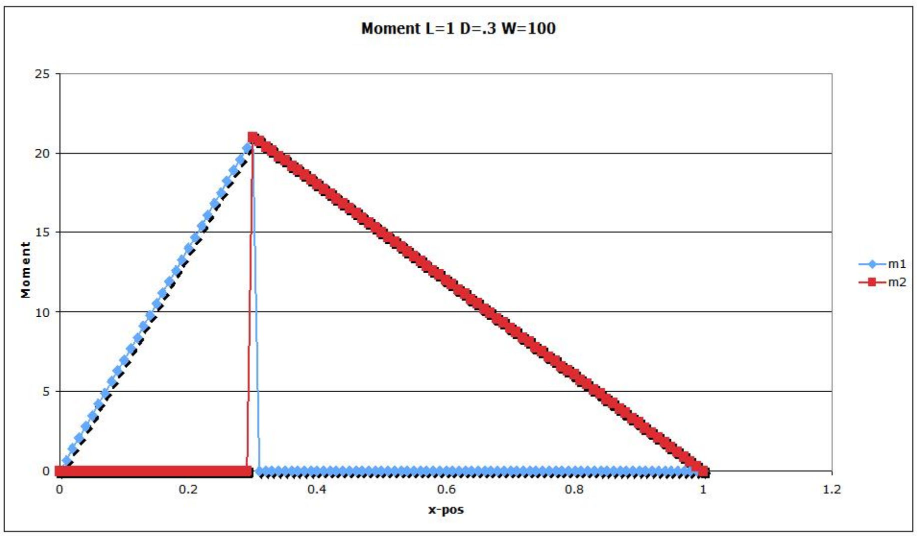

Figure 8: Moment as a function of position.

From these moment equations the deflection of the beam can calculated. This is due to the fact that the stress in the beam is related to the moment over the cross sectional inertia of the beam. In turn the stress can be related to the deflection through young's modulus E and will form the equation

$$y''(x) = \frac{M}{EI} \tag{7}$$

Solving this second order differential equation under certain boundary conditions will give the deflection [2]. In this study two sets of boundary conditions were explored relating to the two most predominate modes of vibration of the beam. In each of those modes, two different differential equations need to be simultaneously solved for the two moment equations. This means that a total of four boundary conditions need to be defined for each mode. The first set, shown shown in figure **??** A, is where the two ends are free to rotate around a fixed position, and that the beam is continuous at D. These conditions can be represented in the equations

The second mode, shown in figure **??** B, has the boundary conditions that the one end is restricted in both translation and rotation, and again the beam is continuous. These conditions can

$$y_1(0) = y_2(L) = 0 \tag{8}$$

$$y_1(D) = y_2(D) \tag{9}$$

$$\frac{dy_1(D)}{dx} = \frac{dy_2(D)}{dx} \tag{10}$$

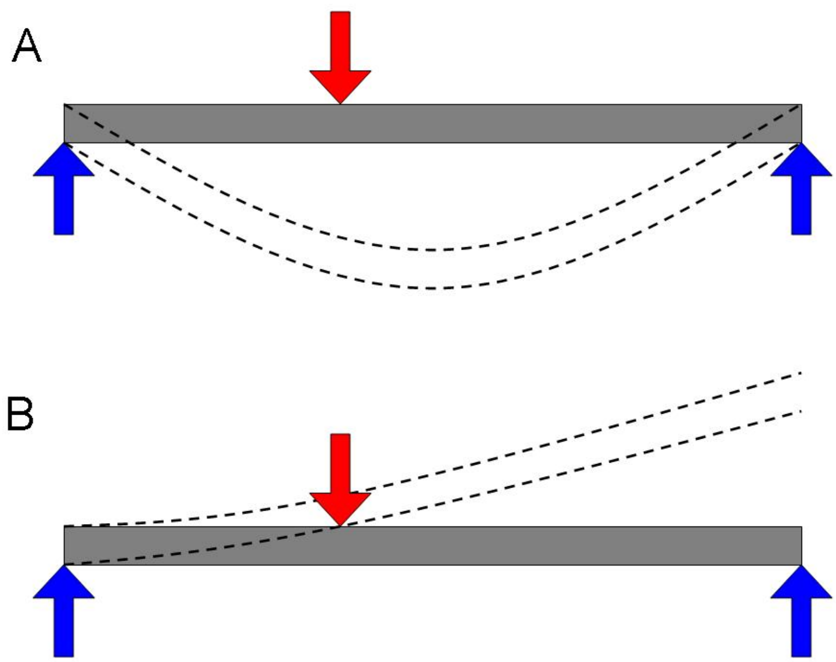

Figure 9: The two vibrational modes.

be represented as

$$g_1(0) = \frac{dg_1(0)}{dx} = 0 \tag{11}$$

$$g_1(D) = g_2(D) \tag{12}$$

$$\frac{dg_1(D)}{dx} = \frac{dg_2(D)}{dx} \tag{13}$$

The solutions to these equations are

The interesting thing to note is that the largest displacement in mode 1 is not at the location of the load. Also it is worthy noting that F_2 is dependent on W in mode 1, and in mode 2 F_2 is independent

$$y_1(x) = \frac{x^3LW - F_2Lx^3 + 3x^2L^2F_2 - 3x^2LWD - D^3W + 3WD^2L - 2F_2L^3}{6EIL} \tag{14}$$

$$y_2(x) = \frac{3F_2L^2x^2 - F_2Lx^3 - 2xF_2L^3 - xD^3W + D^3WL}{6EIL} \tag{15}$$

$$g_1(x) = \frac{Wx^3 - F_2x^3 + 3F_2L + 3WD}{6EI} \tag{16}$$

$$g_2(x) = \frac{-F_2x^3 + 3F_2Lx^2 + 3D^2Wx - D^3W}{6EI} \tag{17}$$

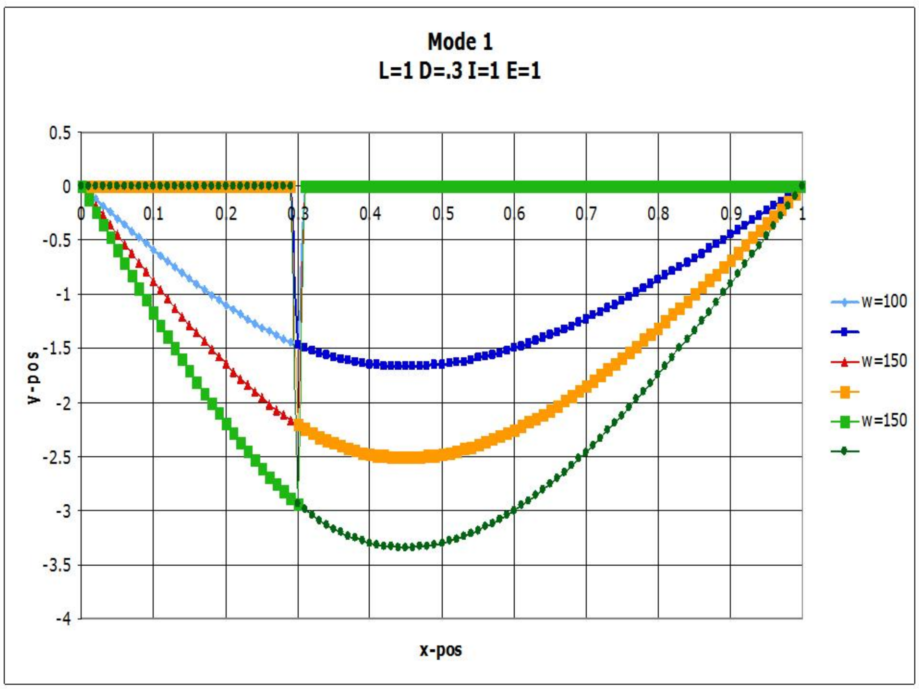

Figure 10: Mode 1 displacement of a beam under three different loads W.

of W. From these displacement calculations the spring constant, K, of the vibrational mode can be calculated by plotting the force vs the displacement at the site of the load. Unfortunately at the time of writing the calculations of the mode 2 K value are not complete and only mode 1 is plotted in figure 11.

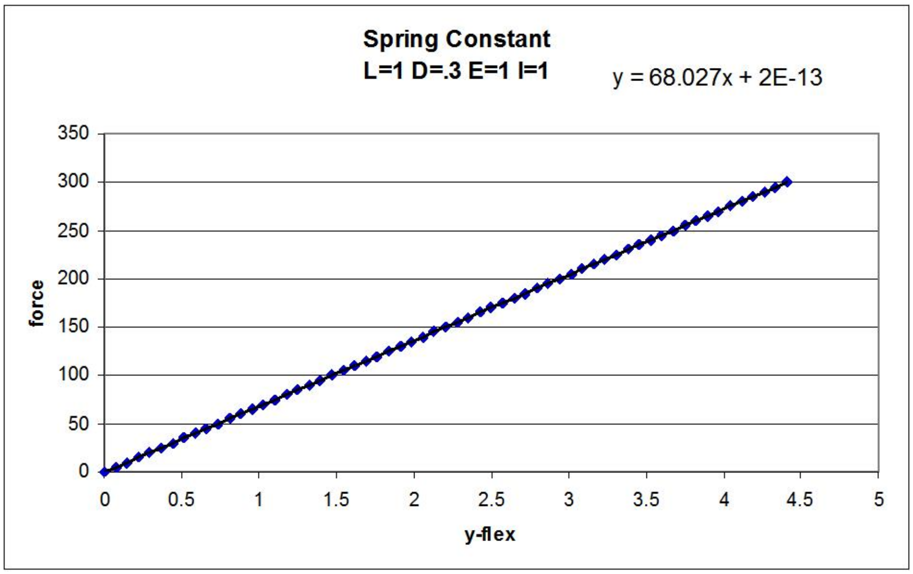

Figure 11: Mode 1 spring constant.

4 Future Research

In the future this research will be continued to include the more complex cases and application of the knowledge learned to the kart. The flexure equations will be used to calculate the resonate frequencies of the beams, and how the beams respond to an off axis weight undergoing an acceleration. The next step is to analyze the chassis as a whole. The method of finite element analysis will be needed to be used for this complex system. Taking measurements on the chassis is a rather difficult task as well. The only available means is to place accelerometers at strategic points on the chassis and see if any discernible resonate oscillations. There is also the possibility of using a scale model to make direct measurements.

References

[1] Taylor, Charles F. $\underline{The Internal - Combustion Engine in Theory and Practice}$. 2nd ed. Vol. 1. Cambridge: The MIT P, 1985.

[2] Harris, Charles O. $\underline{Statics and Strength of Materials}$. New York: John Wiley and Sons, 1982.

[3] "Tru-power Inc. Honda GX160 Performance Curve" http://www.trupower.com/honda/160/160perf.htm