Appendix

Variable Stars in M13.
II. The Red Variables and the Globular Cluster Period–Luminosity Relation

A1. Details on the Individual Red Variables

A finding chart for the M13 red variables is given in Kopacki, Kołaczkowski and Pigulski (2003), hereafter KKP03. Here we give the details of our analyses of their light curves. For each variable we first give a summary of previous work on the star and a general description of its light curve. We especially note any results from the Violat Team because most of their papers – Violat Bordonau, Arranz Heras and Díez Gago (2006) and:

http://casanchi.com/ast/M13_200401.pdf, hereafter VB04,
http://casanchi.com/ast/M13_200402.pdf, hereafter VB05a,
http://casanchi.com/ast/M13_200402b.pdf, hereafter VB05b,
http://casanchi.com/ast/m13a01.pdf, hereafter VB15,
http://casanchi.com/ast/l93801.pdf, hereafter VA06,
http://casanchi.com/ast/rojas.pdf, hereafter VA07a,
http://casanchi.com/ast/rojas02.pdf, hereafter VA07b,
http://casanchi.com/ast/v38.pdf, hereafter VAD05,
http://casanchi.com/ast/v38b.pdf, hereafter VAD06a,
http://casanchi.com/ast/v43.pdf, hereafter VAD06b,
http://casanchi.com/ast/v1101.pdf, hereafter VAD06c,
http://casanchi.com/ast/v1701.pdf, hereafter VAD06d,
http://casanchi.com/ast/v2401.pdf, hereafter VAD06e,
http://casanchi.com/ast/09_L414.pdf, hereafter VB02a,
http://casanchi.com/ast/l414_2.pdf, hereafter VB02b,
http://casanchi.com/ast/l940.pdf, hereafter VB03a,
http://casanchi.com/ast/l96.pdf, hereafter VB03b,
http://casanchi.com/ast/l629.pdf, hereafter VB03c,
http://casanchi.com/ast/fijas.pdf, hereafter VB04a,
http://casanchi.com/ast/v41.pdf, hereafter VB04b,

are not discoverable through the professional literature.

We next discuss the results from the light curves from our photometry, including any periodicities detected. We end by presenting our adopted cycle times ("periods") and variability type – either SRb for those stars with solid evidence of persistent and interacting short periods, SR for other stars for which we found periodicities in their observations, or L-type for stars for which evidence of persistent periodic behavior was not conclusive.

V11

The *V*-band photometry of the Violat Team from 2001 through 2006 shows V11 to have a rather irregular and changeable light curve (VAD06c, VA07a). Fig. 5 of VAD06c shows that during their 2001, 2002 and 2003 observing seasons V11 had fairly coherent cycling with large amplitudes and a characteristic cycle time between 100 d and 120 d. During the 2004 season, they observed a very different light curve, with low amplitude and short variation timescales around 40–50 d. The larger amplitude, more coherent behavior reappeared during 2005, whereas their 2006 data, shown in Fig. 2 and 3 of VA07a, have cycles with maxima of consistent brightness but alternating deep and shallow minima and an amplitude and cycle time similar to that seen in 2004. Unfortunately, the gaps between their annual observing seasons, typically about 200 d, make it difficult to trace any pattern in the modulation of the light curve. The Violat Team also gathered data over a shortened observation period in 2013 (VB15) and V11 appeared to be then in the lower amplitude mode with a cycle time of about 55 d.

The data from the present study, though less complete in terms of coverage, show the same modulated light curve behavior. The Białków 2001 data (KKP03), taken just before the Violat Team's 2001 observations (VAD06c) began, show irregular, low-amplitude variations with a characteristic timescale of 30–40 d. Like the Violat Team's 2004 observations, our 2004 data from MSU and Macalester show that V11 was in its low-amplitude, incoherent mode at that time. The BGSU light curves from 2008–2011 suggest V11 was in a low-amplitude mode in 2008, transitioned from incoherent behavior in 2009 to more coherence in 2010 with greater amplitude variations and more regular cycle times (as shown in Fig. 2 of the main paper) and this behavior continued in 2011.

Attempts over the years to measure a formal period were inconclusive, with the results depending on the length of the time interval considered and the behavior of the light curve during that interval. For example, VAD06c concluded from their 2001–2005 data that V11 has two principle timescales of variation, one of 91–93 d and a second ≈ 45 d, that interfere to create the observed season-to-season modulations. From 2006 data, VA07a again found two timescales of variation, ≈ 87 d and ≈ 48 d, but VB15 found 62.5 d from a single complete cycle in 2013.

Period searches on our data sets consistently show one or more dominant peaks in the power spectrum near 92 d, ranging from 87 d to 98 d for the different sets. This is similar to the periodicity of 91.77 d proposed by Russev and Russeva (1979c) and to that of 92.42 d from Osborn and Fuenmayor (1977, hereafter OF77). We also detected strong power in some data sets at 125 d and 67 d, but little or no power near 45 d. Fitting the CCD data with our best period of 92 d and searching for additional periods among the residuals (pre-whitening), we found significant power at 83 d and 117 d, suggesting that V11 is multi-periodic. A light curve modeled with these periods and appropriate amplitudes using VSTAR has intervals of coherence and incoherence (constructive and destructive interference) similar to the light curve properties described above, though improved data are needed to show this conclusively. Given the evidence for interacting periods we classify V11 as a semi-regular (SRb) type variable with a characteristic timescale of 92 d.

V15

OF77 found V15 to have a low amplitude light curve with a period of 39.23 d based on photographic observations spanning the 1967–1969 observing seasons. Russev and Russeva (1979c) proposed a period of 140.3 d but later showed their observations could also be fit with the OF77 period (Russeva and Russev 1980). We confirm both periods are in their data, but find a stronger signal with 25.6 d. Based on data from his 2004 observing season, spanning 142 d and nearly four complete variation cycles, VB05a deduced a period between 32 d and 39 d, with 31.01 d yielding a reasonable phased light curve, albeit with large scatter relative to the low amplitude. VA07a tracked V15 over five complete cycles during their 2006 observing campaign. The star showed erratic behavior, with the cycles ranging in length from ≈ 22 d to ≈ 49 d. They were unable to find a satisfactory phased light curve, including using their period of most power (43.19 d) and that of OF77. Most recently, VB15 found a period of 72 d from observations covering 122 d in 2013.

Our CCD observations confirm the irregular behavior of the light curve. The results from period searches on our data vary greatly, depending on the data set used and time interval it covered. Specifically, from the BGSU data that cover 2006–2011 we obtain a clear peak in the periodogram at 31.9 d from the *V*-band observations and at 35.0 d, which is related to 31.9 d by the common aliasing factor of one year, in the corresponding I_C-band data. However, the strongest power in the MSU *B*-band data, which covers 2003–2010, is at 139 d with only two relatively weak periodicities seen in the 25–45 d range, both around 30 d. The 2001 Białków observations yielded periodicities of 52.2 d and 20.4 d for the *V* and I_C observations respectively, although visually there is some evidence the *V* value should be halved. Cycle times seem to average about 30 d but individual cycles differ substantially from this value. We tentatively characterize V15 as an SR-type variable.

V17

OF77 found V17 to have a period of 39.14 d over the three observing seasons of 1967–1969. Russev and Russeva (1979a), found these observations plus their own in the period 1962–1977 could be fit with the alias-related period of 43.49 d, but that period appeared to decrease by 0.45 d over their 16 years of coverage. VAD06d presented data for five seasons (2001–2005) taken with the same observing equipment. Their period search on these showed strongest power at 76.5 d and 97.1 d but both produced phased light curves with considerable scatter. Observations with two other telescopes in 2004 and 2005 yielded best periods ranging between 45 d and 83 d, depending on the data set. They suggested the star has an alternating pattern of high and low maxima (and minima) which might be produced by superposition of two periods. This behavior is best seen in their 2005 observations (their

Fig. 12), for which they report a best period of 50.1 d from the 2.5 cycles observed that season alone. However, their observations from 2004 (VB05a) present a very different behavior, with a slow rise taking ≈ 55 d to reach a distinct maximum, followed by a slow decline. Two bumps are seen upon this trend, one preceding and one following the maximum by ≈ 25 d, indicating power is present at shorter timescales as well. VAD06d concluded that the principle period of V17 is about 44 d, but emphasized that the light curve shape and timescale vary from cycle to cycle, either due to changes in the period or interference between multiple periods. The Violat Team data from the 2006 season (VA07a) again show the pattern of alternating high-low cycles with a period of ≈ 43 d over four cycles while in the 2013 season the star presented a single cycle long-period of ≈ 67 d (VB15).

The light curves and periodogram analysis of our data sets give variable results. The Białków V and I_C CCD observations indicate a ≈ 27 d cycle time in 2001. The 2003 MSU B-band CCD observations show a slow decline over > 50 d, followed by a single, slow but high-amplitude cycle of > 120 d in 2004, in agreement with VAD06d. The BGSU V and I_C light curves, particularly for 2009–2011, show a semi-regular cyclicity of ≈ 40 d. A period search on all the BGSU data yields periodogram peaks at the alias-related periods of 42.8 d and 38.3 d. Our combined CCD B data gave the strongest power at 119 d and 127 d, likely reflecting the long cyclicities seen in 2003 and 2004 when the time coverage of the B observations was best. On the other hand, searches on the combined CCD V data gave periods of 25.7 d and 43.3 d while the photographic B data yielded two strong, alias-related, peaks at 39.4 d and 44.2 d. We conclude that, like V11, V17 exhibits episodes of coherence and incoherence that is due to superposition of two or more characteristic periods, probably 43 d and a shorter, ≈ 26 d, one. This 26 d period is seen more clearly in the CCD data after removing the stronger 43 d variations. We classify V17 as a semi-regular SRb type variable.

V18

This star is one of the fainter of the red variables and its variations are of low amplitude. Russeva and Russev (1980) and Welty (1985) published possible periods of 41.25 d and 63.1 d, respectively, from their photographic observations. VB05a presented V-band CCD data taken over 120 nights in the 2004 season. While his measures have significant scatter, low-amplitude variations with a period of 24.7 d were found. The Violat Team observations in 2006 (VA07a) show small (≈ 0.04 mag) fluctuations around $V = 12.40$ mag for 50 d followed by a rise to $V = 12.28$ mag and then 0.04 mag fluctuations about this value for 115 d. Visually the fluctuation cycle averaged 22 d. The VA07a period search yielded periodicities of 13.86 d and 38.03 d, and they suggested the shorter reflects the fluctuations and the longer the cycle of the larger variations. We feel the evidence for this is weak given that some observed plateaus lasted much longer than 38 d. The Team's observations from 2013 (VB15) show a slow decline with little evidence of short-term modulation.

Our CCD observations show hints of short-period variations superposed on a slower, larger-amplitude brightness changes. This is most obvious in the 2009 and 2010 BGSU observations and in the 2001 Białków ones. The former show a distinct period at ≈ 190 d and the latter, while only covering 80 d, is consistent with this as shown in KKP03. The combined blue (*B* and pg) photographic data show only two periods with significant power: 201 d and its one-year alias 130 d. No trustworthy signal is found for a shorter period. We conclude that V18 shows a "long secondary period" (LSP) of about 200 d of the type described by Percy *et al.* (2004) and Wood *et al.* (2004) on which are superposed non-persistent short timescale pulsations. We classify the star as L-type.

V19

Russeva and Russev (1980) and Welty (1985) found, respectively, periods of 44.48 d and a "marginally significant" 32.95 d from their photographic data. The CCD observations of the Violat Team for the 2004 season (VB05a) show large scatter compared to the star's brightness variations. The data were best fit with a 58.78 d period, but power at 32.9 d and 24.7 d periods were also seen in the periodogram. The lack of agreement suggests that all three period searches were compromised by low signal-to-noise combined with limited time coverage or non-optimal cadencing. The Violat Team's better quality observations in the 2006 season cover 193 d and four variation cycles (VA07a). The resulting light curve shows changes in the levels of both maxima and minima and in cycle time, which ranges from 24 d to 57 d. Their period search yielded power at 30.37 d and 52.28 d, and they concluded multiple periods were acting. The team's 2013 data (VB15) showed irregular light curve behavior, similar to that seen in 2006, with a best period of 30.89 d.

Our data sets with suitable time coverage generally confirm the irregularity in the light curve although many of the 2001 Białków I_C images are saturated and could not be used as an independent check on the *V* results. Our period searches consistently yield a strong slow variation near 168–170 d in the combined CCD *B* data, 165 d in the combined *V* and (of less power) 168 d in the combined I_C – which we associate with a long secondary period. There is also evidence for longer periodicities, but the only one that is in common across data sets is ≈ 310 d, which is a one-year alias of 168 d. The short coverage of our observational seasons ($\lesssim 200$ d) makes it impossible to say if the longer possibility should be preferred and we tentatively adopt 168 d as the LSP.

Evidence for a shorter period was mixed. All data sets showed power at some period less than 50 d but the values varied significantly with the most common being around one month. Overall, our results suggest at least two periods are present, an LSP at 168 d and a shorter one that varies significantly around 30 d. We tentatively adopt 30 d for the characteristic cycle time and classify the star as SR-type given our inability to identify multiple short periods.

V20

This star lies 4.′5 from the cluster center and it was not in the field of view of several of the CCD series. In particular, it was not observed in our quality 2001 Białków observations nor in the long-coverage Violat Team observations for the 2006 season. VB05a did publish a light curve for the 2004 season covering 105 d. A period of 43.78 d was derived, but doubling this to 87.56 d gave a smoother light curve having alternating deep and shallow depths of minima. The Team's 2013 data, however, had a best period of 51.69 d (VB15) while published periods from photographic data are 62.26 d (Russeva and Russeva 1980) and ≈ 96.6 d (Welty 1985).

Our light curves show irregularity in brightness level of maxima and minima and in cycle time. The results from period searches on our data were not concordant between the data sets from different passbands and from different telescopes, and often with an inability to decide among several aliases. Peak power was seen at 100 d in the combined B data, at 36.7 d in V and at 67.2 d in I_C. We suggest a cycle time that varies widely around ≈ 40 d and tentatively call V20 SR-type.

V24

The two stars V24 and V39 lie close to each other at the edge of most dense part of the cluster. Consequently, the data for both are rather poor. The light curve for V24 from the Violat Team's extensive observations in 2006 (VA07a) show different levels of maxima and minima with an average cycle time near 50 d. This behavior is also seen in the BGSU observations, but with longer cycle times. The main feature of the 2001 Białków observations is a decrease in brightness over 90 d with fluctuations.

The Russeva, Iliev and Russev (1982) observations yielded a period of 45.34 d, which was confirmed by our period search on their data. The Violat Team observed the star over several summer seasons. Our visual inspection of their published light curves suggests cycles of ≈ 50 d in 2003 and 2004 and ≈ 60 d in 2005 (VAD06e) from 1–2 cycles per season, and irregular variations over three cycles in 2006 (VA07a) with a mean period of 50 ± 7 d. The Team's period analysis found strong signals at about 70 d, 77 d, 87 d and 93 d from their various 2004 and 2005 independent data sets, with 87.35 d producing a reasonably smooth light curve. Their long series of data in 2006 yielded periods of 46.77 d and 78.34 d, though neither resulted in a clean phased light curve, and the authors suggested the presence of multiple periods in superposition (VA07a).

Our period search on the Białków observations gave inconsistent results for the two passbands. Visual inspection of the BGSU data shows clearly-defined cycles in 2009–2010 with semi-regular periodicity around 50 d. The period search found power at two related, relatively short periods – 41.5 d and ≈ 46 d – but no significant power at periods over 72 d. The period with most power in the Osborn-N photographic B data was 59.4 d, which is an alias of 45 d. Based on all period search results we adopt 45 d as the principal cycle time for V24. Pre-whitening with this period yielded possible secondary periods at 95 d and 50 d. We characterize the star as probably a multi-periodic SRb variable similar in many ways to V11 and V17.

V32

This star is listed as a semi-regular variable in Clement (2013). Nevertheless, V32's location in the cluster color–magnitude diagram (see Fig. 1) is where the asymptotic and horizontal branches meet, not near the tip of the red giant branch, so it is not a usual globular cluster red giant variable. It is a cluster member (Cudworth and Monet 1979). With a projected distance from the cluster center of $3\farcm8$, V32 was outside the field of view of most of the CCD studies and we have limited observations: only those published by Russeva *et al.* (1982) and three sets reported here (Welty, BGSU and the short Białków 2014 series). Inspection of the light curves showed low-amplitude variations in which no periodicity could be discerned.

A period search in the range 0.1–2.0 d was done on the well-covered seven-day span of observations in 2014 from Białków. No significant period was found, which rules out short-period ($\leq$ few days) variations. Russeva *et al.* (1982) suggested a period of 21.165 d but we find more power at 34.8 d. Period searches on our longer observation sets showed no single period fits all the data, but all data sets showed significant periods between 22 d and 40 d. We adopt ≈ 33 d as the average, but variable, cycle time and classify the star as a low-amplitude SR-type variable.

V33

The $3\farcm8$ projected distance of V33 from the cluster center meant that it was not observed in several of the CCD series. The available light curves show a small amplitude and irregular behavior with a cycle time around 40 d. Published periods obtained from period searches were 40.37 d (marginally significant) by Welty (1985) from photographic observations over six years and 40.55 d in 2004 (VB05b), 32.52 d and (a better) 62.97 d in 2006 (VA07b) and 50.92 d in 2013 (VB15) by the Violat Team.

Period searches on our data gave results consistent with the published results. A signal near 33 d is seen in all observations sets although not always the period with most power. Other significant periods found in some sets include longer ones near 60 d, 71 d and 89 d as well as the one-year aliases of 33 d of 30 d, 36, 40 d and 45 d. Our best estimate for the characteristic timescale of V33 is about 33 d, with significant variations. We classify the star as SR-type.

V38

The seasonal light curves with coverage over 100 d show irregularity in maxima and minima and evidence of both a cycle time near 40 d and a longer one. The Violat team analyzed their observations from the 2001–2005 seasons along with those of OF77 and KKP03 and concluded a period of about 81 d can fit all these data (VAD05, Violat Bordonau *et al*. 2006). Nevertheless, the Team's analysis of their better-quality 2006 observations showed an irregular behavior with a best period of 48.14 d (VA07b). They suggested interactions by more than one period in V38.

The period searches on our various data sets gave inconsistent results. The BGSU observations showed power at 21 d and 32 d periods and their aliases and these are also seen in most other data sets. On the other hand, longer periods with significant power were also found when searches were done on the combined data series, including periods near 44 d, 53 d, 72 d and 80 d and their multiples. The aggregate results suggest that V38 may be multi-periodic, but better observations are needed to decide for sure. For now we classify the star as SR-type with a characteristic timescale of ≈ 32 d.

V39

This star is one of the brightest in M13 but lies at the edge of the densest part of the cluster and close to V24 which have resulted in fewer and lower quality observations. Our only new observations are the CCD data from BGSU and Białków (but the 2001 I_C images were saturated) plus the Osborn-N photographic measures.

The Violat Team's observations for the 2002 season show a very regular light curve over three and a half cycles that can be fit well with a period of 46.16 d (VB03c). Their observations from 2004 again show a regular light curve, but now with a period of 70.07 d and from only one and a half cycles (VB05b). The Team's observations in the 2006 season covering 200 d (VA07b) give a clearer picture of the light curve behavior: for 80 d the brightness showed irregular rapid – few day – fluctuations followed by a time of more regular variations of changing amplitude and a timescale near 30 d. The period analysis yielded peaks at 27.35 d and (stronger) at 35.30 d, but both gave phased light curves with large scatter and they again suggested multiple periods are present.

Our seasonal light curves with sufficient coverage confirm the irregular behavior. As examples, the 2001 Białków observations show a smoothly varying light curve with a ≈ 60 day timescale while the BGSU data from 2010 show a 70 d period with irregular variations followed by a 100 d window when the light curve was more coherent. Our period searches showed power near 56 d in all data sets but usually with large scatter in the resulting light curves. We classify V39 as a SR-type variable with a variable cycle time averaging about 56 d.

V40

The Violat Team's 2001 and 2002 observations (VB03a) show slow variations with a period analysis giving a best period of 143.68 d but also power at about its 104 d alias. The difference apparently arises from how many cycles are used to link the two years and the seasonal light curves support the shorter value. Their

2004 observations had large scatter and showed weak periodicity at ≈ 28 d and ≈ 48 d, both of which produced poor phased light curves (VB05b). The better 2006 data show a light curve with two cycles of fairly regular variation followed by about 90 d of low-amplitude incoherent behavior after which the larger, more regular cycles reoccur (VA07b). The period analysis on the 2006 data showed three short periodicities ranging from 22 d to 43 d and a longer one at 173.9 d. Phased light curves using the shorter periods all suggested multiple interacting periods. The 2013 observations covered about two cycles and were best fit by a 35.87 d period (VB15).

Inspection of our light curves showed periodic variations with estimated cycle times between 30 d and 50 d in the 1967, 2001, 2004, 2009 and 2010 seasons. The period analysis yielded many periods, depending on the data set, but two occur at some power in all sets: ≈ 33 d and a LSP at ≈ 170 d. As for V19, we are unable to rule out that the true LSP period is longer (periodicities around 315 d are seen in most data sets). We adopt the 33 d as the characteristic cycle time and concur with the Violat Team that there may be interacting multiple periods. Based on the seasonal light curves we classify the star as SR-type.

V41

This star, which has a regular light curve with a 42.5 d period, is discussed in Section 3.4 of the main paper.

V42

V42 is one of the brighter stars in M13, but has a close optical companion which complicates photometry. VB05b observed the star in the 2004 season but acknowledged the measures were compromised. The light curve shows significant scatter with oscillations around 40 d. The period search gave several periods between 8.7 d and 84 d but it was concluded all were false. The 2006 light curve (VA07b) shows oscillations with cycle times ranging from 22 d to 38 d. The period search yielded 23.48 d and 35.99 d, but neither gave a good phased light curve.

Our light curves indicate varying cycle times between 20 d and 60 d. Two periods consistently appear in our data sets, but usually not at maximum power: ≈ 42 d and ≈ 320 d. That at 320 d gives fairly smooth light curves for both the combined V and I_C observations. Considering all the results together, we adopt 40 d as the average, but variable, cycle time that is superposed on a LSP of 320 d. We classify V42 as SR-type.

V43

The Violat Team's 2002 observations (VB03b) show low-amplitude variations with the period search giving a period of 53.12 d over the three cycles covered. The observations for 2004 and 2005 seasons showed large scatter with periods of 35.75 d and 95.8 d found in the 2004 data and 96.9 d and 86.7 d in the 2005 observations (VB05b, VAD06b). The 2006 data (VA07b) cover at least five cycles and show the irregularity of the light curve. Cycle times from 18 d to 40 d are seen with the periodogram showing power at 17.85 d, 26.59 d and (a possibly false) 180.4 d. The 2013 light curve shows a single cycle of about 65 d (VB15).

Our light curves confirm the irregularity of the light curve, including the large variations in cycle time. Our period analysis finds both shorter (20–40 d) and longer (>70 d) periods in all data sets. We are unwilling to adopt even a very uncertain short-period cycle time and classify the star as L-type.

V44 and V45

These two stars are in the central region of the cluster and the observational data are both sparse and uncertain. Also, V44 has a bright, close companion which has likely affected its photometry. Little previous work on their variability has been published. We have both DAOPHOT and ISM-derived magnitudes in V and I_C from the BGSU data set. KKP03 observed the stars but we were unable to transform reliably the flux measures derived from image subtraction to magnitudes.

The light curves suggest variations that are irregular with large changes in the cycle time. Including the results from period searches, our best interpretation is short-term fluctuation – power is seen at ≈ 10.5 d and its multiples – possibly superposed on a longer-period variation of around 120 d. We classify V44 as L-type.

The available V45 light curves show variation but no evidence of periodicity, neither in the light curves nor in the period search results. We classify this star as L-type.

A2. New CCD Photometry of the Red Variables

The individual observations of the M13 red variables from our CCD photometry are available in six machine-readable tables. The columns of each table give the source of the observation, the heliocentric Julian date, and the magnitude and its error for each of the variables observed on that image. The errors are from the CCD photometric reductions and do not include possible zero point errors of up to 0.02 mag. The data sources are identified by:

Bial-01 – from images with the Wrocław Observatory Białków Station 0.6-m reflector in 2001

Bial-14 – from images with the Wrocław Observatory Białków Station 0.6-m reflector in 2014

BGSU – from images with the Bowling Green State University 0.5-m reflector in 2006–2011

Macal. – from images with the Macalester College 0.4-m reflector in 2004

MSU – from images with the Michigan State University 0.6-m reflector in 2003–2010

The B-band data are given in Table A1a for V11–V33 and Table A1b for V38–V45. The V data are given in Tables A2a and A2b and the I_C data in Tables A3a and A3b. We note that many of the Bial-01 I_C-band images of V19, V38, V40 and V42 were saturated and, while listed in Tables A3a and A3b, those data should probably be disregarded.

A3. Additional Photometry of the Red Variables

Several sets of observations of the M13 red variables used in this study had been utilized in previous investigations. In two cases the individual observations were published (Demers 1971[1], Pike and Meston 1977[2]), but in others the observations were not included in the resulting papers. The following sets of unpublished observations were kindly provided us by those papers' authors.

Russev: The photographic observations of Russev and collaborators (Russev 1973, Russev and Russeva 1979a, 1979b, 1979c, Russeva and Russev 1980, Russeva, Illiev and Russev 1982) provided by R. Russev are given in Table B1. The table columns list for each night the mean heliocentric Julian Date, the mean magnitudes for the variables and numbers of plates used for each mean. The HJD 2437790–2441092 observations are photographic magnitudes from plates taken with the Moscow 70-cm AZT-2 reflector and the remainder of the observations are photographic B ones from plates taken with the Belogradchik (Bulgaria) 60-cm reflector. Russev provided nightly means (and some small corrections) to replace the previously published results for V11 and V15 so the data for these stars would be consistent with the nightly means provided for the other variables from the same plates. The error of a listed magnitude is about 0.08 mag.

Welty: The unpublished photographic V and B observations of Welty (1985) from Yerkes 1-m reflector plates, provided us by Dr. Welty, are given in Table B2. The first line of the table is the sole V entry while the other lines are the B observations. Comparison of the different magnitudes for a given star on the same night showed the error of an individual magnitude is about 0.05 mag.

Osborn-N: The unpublished photographic observations of Osborn (2000) from U.S. Naval Observatory Flagstaff Station 1.55-m reflector plates are given in Table B3. Table columns are the magnitude type (U, B or V), the heliocentric Julian Date of mid-exposure of the plate and the magnitudes for those variables measured on that plate. Values with colons have errors of about 0.10 mag, the others about 0.05 mag. The present data supersede the magnitudes published for some of the plates by OF77.

Osborn-L: The unpublished photoelectric observations of Osborn (2000) made at Lowell Observatory are given in Table B4. The table gives the variable, the

[1]A review of the observing log book revealed that Demer's first observation was made on 2439622.825 not the published 2439625.825.

[2]Pike and Meson's published Julian Dates have been corrected as per Wehlau and Bohlender (1982).

heliocentric Julian Date for the mid-time of the observation, and the $UBV(RI)_C$ photometry. The error of a given magnitude – not color – is 0.03 mag.

A4. Number of Observations per Observing Season

Table 1 below gives the observations available for each observing season in the different passbands for the M13 red variables. The first two columns of Table 1 give the source of the observations (see Sections 2 and 3) and the observation method: CCD, photoelectric (pe) or photographic (pg). This is followed by the observing season and number of nights, and in parentheses the number of observations, for each variable that season. The photographic data are, of course, less reliable than the CCD measures, and the *U* observations are particularly uncertain.

Table 1

Ref	Obs	Year	V11	V15	V17	V18	V19	V20	V24	V32	V33	V38	V39	V40	V41	V42	V43	V44	V45
Seasonal observations of M13 red variables in *U*																			
Demers	pg	1967	5(5)																
Osborn-N	pg	1967	1(1)	1(1)	1(1)	1(1)	1(1)	1(1)	1(1)	1(1)	1(1)	1(1)	1(1)	1(1)					
Osborn-L	pe	1983		2(2)	4(4)	6(10)		10(17)			12(24)	6(8)							
Osborn-L	pe	1991									1(1)								
Seasonal observations of M13 red variables in *B*																			
Russev	pg	1962	2(2)	2(2)	2(3)		2(3)												
Russev	pg	1963	8(16)	8(16)	8(15)		9(13)												
Russev	pg	1965	1(1)	1(1)	1(1)		1(1)												
Demers	pg	1967	11(11)																
Osborn-N	pg	1967	21(29)	20(27)	20(27)	10(13)	10(13)	10(13)	10(13)		10(13)	21(28)	10(13)	10(13)					
Osborn-N	pg	1968	13(22)	11(20)	11(20)	10(19)	10(19)	10(19)	10(19)		10(19)	11(20)	10(19)	10(19)					
Osborn-N	pg	1969	7(8)	7(8)	7(8)	7(8)	7(8)	7(8)	7(8)		7(8)	7(8)	7(8)	7(8)					
Pike-Mest.	pg	1971	6(6)					6(6)											
Russev	pg	1971	2(3)	2(3)	2(3)		2(3)												
Russev	pg	1974	10(14)	12(16)	12(14)	10(13)	10(12)	11(13)	7(8)	7(7)									
Osborn-N	pg	1976	4(8)	3(7)	4(8)	3(7)	4(8)	4(8)	3(6)		3(7)	3(7)	3(7)	3(7)					
Russev	pg	1976	2(4)	2(2)	2(4)	2(4)	2(4)	2(3)	2(3)	2(3)									
Russev	pg	1977	8(19)	8(19)	8(19)	8(19)	8(19)	8(19)	8(19)	8(14)									
Russev	pg	1978	3(6)	3(6)	3(6)	3(5)	3(6)	3(6)	2(5)	2(4)									
Welty	pg	1978	8(18)	8(18)	8(18)	8(18)	8(18)	8(18)		8(18)	8(18)	8(18)			8(18)		8(18)		
Russev	pg	1979	5(14)	5(13)	5(14)	5(13)	5(14)	5(14)	5(15)	4(12)									
Welty	pg	1979	8(16)	8(16)	8(16)	8(16)	8(16)	8(16)		8(14)	8(16)	8(16)			8(16)		8(16)		
Welty	pg	1980	15(22)	15(22)	15(22)	15(22)	15(22)	15(22)		14(21)	15(22)	15(22)			14(21)		15(22)		
Russev	pg	1981	3(4)	3(5)	2(3)	3(4)	3(4)	2(3)	6(8)	6(8)									
Welty	pg	1982	3(6)	3(6)	3(6)	3(6)	3(6)	3(6)		3(5)	3(6)		3(6)		3(6)		3(6)		
Osborn-N	pg	1983	9(18)	9(18)	9(18)	9(18)	9(18)	9(18)	8(16)		9(18)	9(18)	9(18)	9(18)					
Osborn-L	pe	1983		2(2)	4(5)	7(11)		13(23)			18(30)	10(15)							
Welty	pg	1983				1(1)		1(1)		1(1)	1(1)								
Welty	pg	1984				1(1)		1(1)		1(1)	1(1)								
Osborn-L	pe	1991				1(1)		2(2)			2(2)								
MSU	CCD	2003	6(10)	6(10)	6(10)		6(10)					6(9)		6(10)	6(10)				
MSU	CCD	2004	10(12)	10(12)	10(12)		10(12)					10(12)		10(12	8(9)				
Macalester	CCD	2004	13(13)	13(13)	13(13)		15(15)					13(13)		13(13)	11(11)				
MSU	CCD	2006	12(14)	10(12)	12(14)		12(14)					12(14)		12(14)	12(12)				
MSU	CCD	2008	5(11)	5(9)	5(11)		5(11)					5(11)		5(11)	5(11)				
MSU	CCD	2009	4(4)	4(4)	4(4)		4(4)					4(4)		4(4)	4(4)				
MSU	CCD	2010	12(20)	12(19)	12(20)		12(20)					12(20)		12(20)	10(15)				
Białków	CCD	2014	6(193)	6(182)	6(193)	6(183)	6(196)	6(187)	6(189)	6(185)	6(186)	6(193)	6(187)	6(180)	6(170)	6(195)	6(190)		

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Concluded

Ref	Obs	Year	V11	V15	V17	V18	V19	V20	V24	V32	V33	V38	V39	V40	V41	V42	V43	V44	V45
Seasonal observations of M13 red variables in V																			
Demers	pg	1967	12(12)																
Pike-Mest.	pg	1971	6(6)					6(6)											
Osborn-N	pg	1976	4(7)	3(6)	4(8)	3(6)	4(8)	4(8)	3(6)		3(6)	3(6)	3(6)	3(6)					
Welty	pg	1976	1(1)	1(1)	1(1)	1(1)	1(1)	1(1)		1(1)	1(1)	1(1)			1(1)		1(1)		
Osborn-N	pg	1983	9(18)	9(18)	9(18)	9(18)	9(18)	9(18)	9(18)		9(18)	9(18)	9(18)	9(18)					
Osborn-L	pe	1983		2(2)	4(5)	7(11)		13(23)			18(31)	10(15)							
Osborn-L	pe	1991				1(1)		2(2)			2(2)								
Białków	CCD	2001	23(331)	23(333)	23(337)	20(279)	23(323)		23(336)			21(292)	23(336)	22(317)	22(326)	23(342)	21(284)		
MSU	CCD	2003	1(1)	1(1)	1(1)		1(1)					1(1)		1(1)	6(10)				
MSU	CCD	2004	3(3)	3(3)	3(3)		3(3)					3(3)		3(3)	8(9)				
Macalester	CCD	2004	4(4)	4(4)	4(4)		4(4)					4(4)		4(4)	15(15)				
MSU	CCD	2006	3(3)	3(3)	3(3)		3(3)					3(3)		3(3)	12(12)				
BGSU	CCD	2006	5(5)	5(5)	5(5)	5(5)	5(5)	5(5)	5(5)	5(5)	5(5)	5(5)	5(5)	5(5)	5(5))	5(5))	5(5))	5(5)	5(5)
BGSU	CCD	2007	18(18)	18(18)	18(18)	18(18)	18(18)	18(18)	18(18)	17(17)	18(18)	18(18)	18(18)	18(18)	18(18)	18(18)	18(18)	17(17)	17(17)
MSU	CCD	2008	2(6)	2(6)	2(6)		2(6)					2(6)		2(6)	6(12)				
MSU	CCD	2009	1(1)	1(1)	1(1)							1(1)		1(1)	1(1)				
BGSU	CCD	2009	28(28)	28(28)	28(28)	28(28)	28(28)	28(28)	28(28)	28(28)	28(28)	28(28)	28(28)	28(28)	29(29)	29(29)	28(28)	23(23)	23(23)
MSU	CCD	2010	4(5)	4(5))	4(5)		4(5)					4(5)		4(5)	9(15)				
BGSU	CCD	2010	27(28)	27(28)	26(27)	26(27)	26(27)	27(28)	26(27)	26(27)	27(28)	27(28)	27(28)	27(28)	27(28)	27(28)	27(28)	25(26)	25(26)
BGSU	CCD	2011	15(15)	15(15)	15(15)	15(15)	15(15)	15(15)	15(15)	15(15)	15(15)	14(14)	15(15)	15(15)	15(15)	15(15)	14(14)	12(12)	12(12)
Białków	CCD	2014	7(247)	7(224)	7(234)	7(233)	7(240)	7(233)	6(246)	6(234)	7(237)	7(239)	7(243)	7(225)	7(233)	7(227)	7(234)		
Seasonal observations of M13 red variables in I_C																			
Osborn-L	pe	1991				1(1)		2(2)			3(3)								
Białków	CCD	2001	23(307)	23(311)	23(312)	20(254)	23(307)		23(307)			21(275)	23(313)	23(309)	23(312)	23(304)	21(273)		
BGSU	CCD	2006	6(6)	6(6)	6(6)	6(6)	6(6)	6(6)	6(6)	6(6)	6(6)	6(6)	6(6)	6(6)	6(6)	7(7)	6(6)	7(7)	7(7)
BGSU	CCD	2007	22(22)	22(22)	22(22)	22(22)	22(22)	22(22)	22(22)	21(21)	22(22)	22(22)	22(22)	22(22)	22(22))	22(22)	22(22)	22(22)	22(22))
BGSU	CCD	2009	29(29)	29(29)	29(29)	29(29)	28(28)	28(28)	28(28)	29(29)	29(29)	29(29)	28(28)	29(29)	29(28)	29(29)	28(29)	29(29)	28(28)
BGSU	CCD	2010	26(27)	26(27)	26(27)	26(27)	26(27)	26(27)	26(27)	26(27)	26(27)	26(27)	26(27)	26(27)	26(27)	26(27)	26(27)	27(28)	26(27)
BGSU	CCD	2011	14(14)	14(14)	14(14)	14(14)	14(14)	14(14)	14(14)	14(14)	15(15)	14(14)	14(14)	15(15)	15(15)	14(14)	14(14)	15(15)	15(15)
Białków	CCD	2014	7(237)	7(206)	7(219)	7(231)	7(239)	7(224)	7(230)	7(224)	7(231)	7(237)	7(231)	7(234)	7(231)	7(238)	7(232)		